<u>DEF</u> Let $\mathbf{u}$ and $\mathbf{v}$ be vectors in $\mathbb{R}^n$. The ______________________________ of $\mathbf{u}$ and $\mathbf{v}$, denoted $\mathbf{u} \cdot \mathbf{v}$ is defined as

$$\mathbf{u} \cdot \mathbf{v} = \mathbf{u}^T \mathbf{v} = \begin{bmatrix} u_1 & u_2 & u_3 & \cdots & u_n \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_n \end{bmatrix} =$$

<u>NOTE</u> The result of an inner product is a ____________ .

<u>EX</u> Given $\mathbf{u} = \begin{bmatrix} 3 \\ -4 \\ 0 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$, compute

(a). $\mathbf{u} \cdot \mathbf{v}$

(b). $\mathbf{v} \cdot \mathbf{u}$

(c). $\mathbf{u} \cdot \mathbf{u}$

<u>THEOREM</u> Properties of Inner Product

(a). $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$

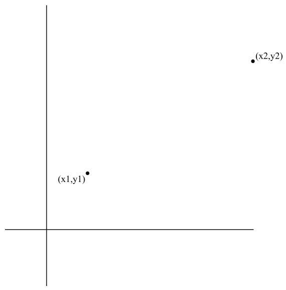
(b). $(\mathbf{u} + \mathbf{v}) \cdot \mathbf{w} = \mathbf{u} \cdot \mathbf{w} + \mathbf{v} \cdot \mathbf{w}$

(c). $(c\mathbf{u}) \cdot \mathbf{v} = c(\mathbf{u}) \cdot \mathbf{v} = \mathbf{u} \cdot (c\mathbf{v})$

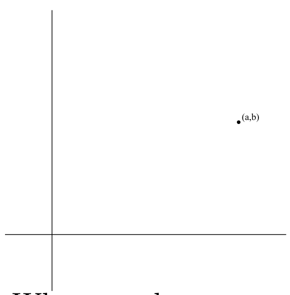
(d). $\mathbf{u} \cdot \mathbf{u} \geq \mathbf{0}$

(e). $\mathbf{u} \cdot \mathbf{u} = \mathbf{0}$ iff $\mathbf{u} = \mathbf{0}$

What is the distance between two points (x_1, y_1) and (x_2, y_2) in $\mathbb{R}^2$?



Suppose the vector $\mathbf{v} = \begin{bmatrix} a \\ b \end{bmatrix}$ in $\mathbb{R}^2$ is in standard position. What is the length of the vector $\mathbf{v}$?



What are the corresponding formulas in $\mathbb{R}^3$?

Extend these ideas to $\mathbb{R}^n$:

<u>Def</u> Let $\mathbf{v}$ be a vector in $\mathbb{R}^n$. The ______________________________ of $\mathbf{v}$, denoted $\|\mathbf{v}\|$ (or $|\mathbf{v}|$), is the nonnegative scalar defined by

$\|\mathbf{v}\| =$

<u>Theorem</u> For any scalar c, $\|c\mathbf{v}\| = |c|\|\mathbf{v}\|$

<u>Proof</u>

<u>Def</u> A __________ vector is a vector with a length of one.

<u>Ex</u> Given $\mathbf{v} = \begin{bmatrix} 2 \\ -1 \\ 2 \\ 0 \end{bmatrix}$, find a unit vector $\mathbf{u}$ in the same direction as $\mathbf{v}$.

Step 1. Find the length of $\mathbf{v}$.

Step 2. Divide $\mathbf{v}$ by its length $\|\mathbf{v}\|$, i.e. $\mathbf{u} = \dfrac{\mathbf{v}}{\|v\|}$.

$\mathbf{u}$ points in the same direction because ____

$\mathbf{u}$ has a length of 1 since ____

<u>Note</u> The process of finding a unit vector in the same direction as $\mathbf{v}$ is called ____________ $\mathbf{v}$.

Recall, that a basis for a vector space (or subspace) is not unique.

<u>Ex</u> Let W be the subspace of $\mathbb{R}^3$ that is spanned by $\mathcal{B}_1 = \left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

Find a basis for W that contains only unit vectors.