Let A be an $n \times n$ square matrix. Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be the linear transformation defined as $T(\mathbf{x}) = A\mathbf{x}$.
Then the following statements are equivalent (i.e. they are either all true or all false).

Fill in the blanks so that the statements are equivalent. Indicate which previous statement(s) and theorem/definition (including page numbers in book) imply each statement.
[Hint: One or more of the previous statements in the IMT will likely be the "if" part of the appropriate theorem to use.]

1. A is an invertible matrix. [To complete the circle, explain why #13 $\Rightarrow$ #1.]

2. There is an $n \times n$ matrix D such that $AD =$ _____.
By (1), A^{-1} exists and let $D = A^{-1}$ in the definition of inverse (Sec. 2.2, p. 103).

3. There is an $n \times n$ matrix C such that $CA =$ _____.

4. The equation $A\mathbf{x} = \mathbf{0}$ has only _____ solution.
By (1) and Theorem 5 (Sec. 2.2, p. 104) the equation $A\mathbf{x} = \mathbf{0}$ has the unique solution $\mathbf{x} = A^{-1}\mathbf{0} = \mathbf{0}$.

5. The columns of A are _____ .
By (4) and Statement (i.e. Theorem) (Sec. 1.7, p. 57).

6. For each $\mathbf{b}$ in $\mathbb{R}^n$, the equation $A\mathbf{x} = \mathbf{b}$ has _____ solution.

7. The columns of A span _____ .

8. A has _____ pivot positions.

9. A is row equivalent to _____

10. A^T is an _____ matrix.

11. T maps $\mathbb{R}^n$ _____ $\mathbb{R}^n$.

12. T is a _____ transformation.

13. $\det A \neq 0$.