Theorem Properties of Matrix Addition and Scalar Multiplication

Let A, B, and C be matrices of the same size and let r, s be scalars. Then

(a). $A + B = B + A$ commutative

(b). $(A + B) + C = A + (B + C)$ associative

(c). $A + 0 = A$ additive identity

(d). $r(A + B) = rA + rB$ distributive

(e). $(r + s)A = rA + sA$ distributive

(f). $r(sA) = (rs)A$ associative

Proof of **(d).**

Theorem Properties of Matrix Multiplication

Let A be an $m \times n$ matrix and let B and C be matrices whose sizes make indicated products and sums defined. Let r be a scalar. Then

(a). $A(BC) = (AB)C$ associative

(b). $A(B + C) = AB + AC$ left distributive

(c). $(B + C)A = BA + CA$ right distributive

(d). $r(AB) = (rA)B = A(rB)$ scalar associative/commutative

(e). $I_mA = A = AI_n$ multiplicative identity

Notes:

Proof of **(a).**

Proof of **(e).**

<u>THEOREM</u> Properties of Transposed Matrices

Let A and B be matrices whose sizes make indicated products and sums defined. Let r be a scalar. Then

(a). $(A^T)^T = A$

(b). $(A + B)^T = A^T + B^T$

(c). $(rA)^T = rA^T$

(d). $(AB)^T = B^T A^T$

Proof of **(a).**

Proof of **(d).**