A matrix with only one column is called a ___(column) vector___ and denoted $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$ (i.e. $n \times 1$ matrix)

Ex:

The entries can be written as ___an ordered list $(u_1, u_2, \ldots, u_n)$___ called an n-tuple.

Ex:

___$\mathbb{R}^n$ (i.e. n-dimensional space)___ is the set of all n-tuples where each entry is in ___$\mathbb{R}$___.

Ex: $n = 2$: (u_1, u_2) is an ___ordered pair___ and defines a point in ___$\mathbb{R}^2$ (i.e. the plane)___.

Ex: $n = 3$: (u_1, u_2, u_3) is an ___ordered triple___ and defines a point in ___$\mathbb{R}^3$ (i.e. 3D-space)___.

Graphical Representations of vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$.

$\mathbf{u} = \begin{bmatrix} a \\ b \end{bmatrix}$ *can* be represented by a line segment w/ an arrow from the origin to the point (a, b). But it can be drawn anywhere in the plane with the same direction and length. Similar for $\mathbb{R}^3$.

Addition of two vectors: Add corresponding entries $\mathbf{u}+\mathbf{v}=\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}+\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}=\begin{bmatrix} u_1+v_1 \\ u_2+v_2 \\ \vdots \\ u_n+v_n \end{bmatrix}$

Examples and Graphical Representations

Scalar Multiplication ($c \in \mathbb{R}$ is a scalar): Multiply each element by c. $c\mathbf{u}=c\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}=\begin{bmatrix} cu_1 \\ cu_2 \\ \vdots \\ cu_n \end{bmatrix}$

Examples and Graphical Representations

Properties for vectors $\mathbf{u}, \mathbf{v}$, and $\mathbf{w}$ in $\mathbb{R}^n$ and scalars $c, d \in \mathbb{R}$.

1. $\mathbf{u}+\mathbf{v}=\mathbf{v}+\mathbf{u}$
2. $(\mathbf{u}+\mathbf{v})+\mathbf{w}=\mathbf{u}+(\mathbf{v}+\mathbf{w})$
3. $\mathbf{u}+\mathbf{0}=\mathbf{0}+\mathbf{u}=\mathbf{u}$
4. $\mathbf{u}+(-\mathbf{u})=-\mathbf{u}+\mathbf{u}=\mathbf{0}$
5. $c(\mathbf{u}+\mathbf{v})=c\mathbf{u}+c\mathbf{v}$
6. $(c+d)\mathbf{u}=c\mathbf{u}+d\mathbf{u}$
7. $c(d\mathbf{u})=(cd)\mathbf{u}$
8. $1\mathbf{u}=\mathbf{u}$