Name: Key

Math 362 Linear Algebra – Crawford

Quiz 4

08 November 2017

Books and notes (in any form) are not are allowed. You may use a calculator – but you must clearly show your set-up for the problem. Please also indicate when you use the matrix functions on the calculator. Show all other work for credit. ***Good luck!*** [Note: Each quiz score will be scaled to 15 points after grading.]

1. (2 pts) If the equation $C\mathbf{u} = \mathbf{v}$ has more than one solution for some $\mathbf{v}$ in $\mathbb{R}^n$, can the columns of the $n \times n$ matrix C span $\mathbb{R}^n$? (***Briefly***) Why or why not?

$C\vec{u} = \vec{v}$ does not have a unique sol^{n}.

$\Rightarrow$ All statements in IMT are false for C.

$\therefore$ The columns of C do not span $\mathbb{R}^n$.

2. (2 pts) Can a square matrix with two identical columns be invertible? (***Briefly***) Why or why not?

If 2 columns are identical, then the columns are not linearly dependent.

$\Rightarrow$ All statements of the IMT are false.

$\therefore$ The matrix is not invertible.

3. (2 pts) If A is a 6×6 matrix and the equation $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b}$ in $\mathbb{R}^6$, is it possible that for some $\mathbf{b}$, the equation $A\mathbf{x} = \mathbf{b}$ has more than one solution? (***Briefly***) Why or why not?

$A\vec{x} = b$ is consistent for all $\vec{b}$

$\Rightarrow$ Columns of A span $\mathbb{R}^6$

$\Rightarrow$ All statements of the IMT are true.

$\Rightarrow$ $A\vec{x} = \vec{b}$ has a unique sol^{n} for all $\vec{b}$.

4. (4 pts) Let W be the set of all vectors of the form $\begin{bmatrix} 2a-3b \\ a+b+2c \\ 0 \\ -3b+c \end{bmatrix}$, where a, b, and c are arbitrary real numbers.

Find a set S that spans W or give an example to show that W is not a vector space.

$$\begin{bmatrix} 2a-3b \\ a+b+2c \\ 0 \\ -3b+c \end{bmatrix} = a\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + b\begin{bmatrix} -3 \\ 1 \\ 0 \\ 3 \end{bmatrix} + c\begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

$$\boxed{S = \left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}}$$

5. (5 pts) Note: $M_{m\times n}$ is the set of all $m \times n$ matrices. $M_{m\times n}$ is also a vector space under addition of matrices and multiplication by a scalar.

Let F be a fixed 3×2 matrix, and let H be the set of all matrices A in $M_{2\times 4}$ with the property that $FA = 0$ (the zero matrix in $M_{3\times 4}$). Determine whether H is a subspace of $M_{2\times 4}$. Show work to justify your answer.

Note: $H = \{A \text{ in } M_{2\times 4} \mid FA = 0\}$

0. $H \subset M_{2\times 4}$ since the matrices A come from $M_{2\times 4}$.

1. Let $A = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{=\vec{0} \in M_{2\times 4}}$. Then $FA = 0$, so $\vec{0} \in H$.

2. Let $A, B \in H$. [Show $A+B \in H$]

Then $FA = 0$ and $FB = 0$ [Show $F(A+B) = 0$]

$F(A+B) = FA + FB$
$= 0 + 0$ $\Rightarrow$ $A+B \in H$ ie H is closed under addition
$= 0$

3. Let $A \in H$ and c be a scalar. [Show $cA \in H$ ie Show $F(cA) = 0$]
Then $FA = 0$.
$F(cA) = c(FA) = c0 = 0 \Rightarrow cA \in H$ ie H is closed under scalar multiplication