Name: Key **Quiz 3**

Math 362 Linear Algebra – Crawford **25 October 2017**

Books and notes (in any form) are not are allowed. You may use a calculator – but you must clearly show your set-up for the problem. Please also indicate when you use the matrix functions on the calculator. Show all other work for credit. ***Good luck!*** [Note: Each quiz score will be scaled to 15 points after grading.]

1. (5 pts) Given the following system of equations, use Cramer's Rule to solve for x_3 only.

$$\begin{array}{rcrcrcr} 2x_1 & + & x_2 & + & x_3 & = & 4 \\ -x_1 & + & & & 2x_3 & = & 2 \\ 3x_1 & + & x_2 & + & 3x_3 & = & -2 \end{array}$$

$$X_3 = \frac{\begin{vmatrix} 2 & 1 & 4 \\ -1 & 0 & 2 \\ 3 & 1 & -2 \end{vmatrix} \begin{matrix} 2 & 1 \\ -1 & 0 \\ 3 & 1 \end{matrix}}{\begin{vmatrix} 2 & 1 & 1 \\ -1 & 0 & 2 \\ 3 & 1 & 3 \end{vmatrix} \begin{matrix} 2 & 1 \\ -1 & 0 \\ 3 & 1 \end{matrix}} = \frac{0+6+(-4)-0-4-2}{0+6+(-1)-0-4-(-3)}$$

$$= \frac{-4}{4} = \boxed{-1}$$

2. (3 pts) Given the following determinant: $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 4,$ find the following determinant.

$$\begin{vmatrix} d & e & f \\ 5a & 5b & 5c \\ g & h & i \end{vmatrix}$$

① $R_1 \Leftrightarrow R_2$ $\Rightarrow \det(B_1) = -\det A = -4$
Row Interchange

② $5R_2 \to R_2$
Row Scaling $\Rightarrow$ $\det(B_2) = 5 \det B_1$

$= 5(-4)$

$= \boxed{-20}$

3. (4 pts) Let U be a square matrix such that $U^T U = I$. Show that $\det U = \pm 1$.

$\det(u^T u) = \det(I)$

$\det(u^T)\det(u) = 1$ by properties of determinant

$\det(u)\det(u) = 1$ since $\det(u^T) = \det(u)$

$(\det u)^2 = 1$

$\Rightarrow \boxed{\det(u) = \pm 1}$

4. (3 pts) Determine if the following statements are True or False. [No explanation necessary.]

(a). $\det(AB) = \det(A)\det(B)$ for all matrices A and B for which the product AB is defined

$\boxed{\text{False.}}$ A and B must be <u>square</u> matrices for det A + det B to exist

<u>Counter-example</u> $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 0 & 1 \end{bmatrix}$ $AB = \begin{bmatrix} 5 & 1 \\ 2 & 1 \end{bmatrix}$

2 x 3 2x3

$\det(AB) = 5 - 2 = 3$
But det A + det B are not defined

(b). Suppose A is a square matrix such that $\det A^3 = 0$, then A is not invertible.

$\boxed{\text{True}}$

$\det(A^3) = (\det A)^3 = 0$

$\Rightarrow \det A = 0$

$\Rightarrow$ A is not invertible