Name: Key

Math 362 Linear Algebra – Crawford

Quiz 2

27 September 2017

Books and notes (in any form) are <u>not</u> are allowed. You may use a calculator – but you must clearly show your set-up for the problem. Please also indicate when you use the matrix functions on the calculator. Show all other work for credit. ***Good luck!*** [Note: Each quiz score will be scaled to 15 points after grading.]

1. (6 pts) Let $A = \begin{bmatrix} 1 & -2 \\ 0 & 4 \\ 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 & -1 \\ 2 & 3 & 4 \end{bmatrix}$.

(a). Compute $(2I_3)A$ or explain why it is undefined.

$$\underbrace{2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{3\times 3}\underbrace{\begin{bmatrix} 1 & -2 \\ 0 & 4 \\ 3 & -1 \end{bmatrix}}_{3\times 2} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 0 & 4 \\ 3 & -1 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 2 & -4 \\ 0 & 8 \\ 6 & -2 \end{bmatrix}}$$

(b). Compute $B^T - 2A$ or explain why it is undefined.

$$\begin{bmatrix} 2 & 2 \\ 0 & 3 \\ -1 & 4 \end{bmatrix} - 2\begin{bmatrix} 1 & -2 \\ 0 & 4 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 2-2 & 2+4 \\ 0-0 & 3-8 \\ -1-6 & 4+2 \end{bmatrix} = \boxed{\begin{bmatrix} 0 & 6 \\ 0 & -5 \\ -7 & 6 \end{bmatrix}}$$

2. (3 pts) Let $B = \begin{bmatrix} 2 & -1 \\ -6 & 3 \end{bmatrix}$. Construct a 2×2 matrix A that contains no zero entries, such that AB is the zero matrix.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -6 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \left.\begin{matrix} 2a - 6b = 0 \\ -a + 3b = 0 \end{matrix}\right\} \Rightarrow \underline{a = 3b} \qquad \begin{matrix} 2c - 6d = 0 \\ -c + 3d = 0 \end{matrix} \Rightarrow \underline{c = 3d}$$

$$\boxed{A = \begin{bmatrix} 3 & 1 \\ 12 & 4 \end{bmatrix} \text{ is } \underline{\text{one}} \text{ possible option}}$$

<u>OR</u> note that for <u>both</u> col^{n}s of B, the 2nd entry is −3 times the 1st entry

So the <u>rows</u> of A must have the 1st entry be 3 times the 2nd entry.

3. (7 pts) Given $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -4 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -2 \\ 2 \\ -3 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 2 \\ h \\ 4 \end{bmatrix}$.

For $A = [\vec{v}_1\ \vec{v}_2\ \vec{v}_3]$, when does $A\vec{x} = \vec{0}$ have a nontrivial sol^{n}.

(a). Find all value(s) of h for which the vectors are linearly *dependent*. Show work to justify your answer.

$$\left[\begin{array}{ccc|c} 1 & -2 & 2 & 0 \\ -4 & 2 & h & 0 \\ 2 & -3 & 4 & 0 \end{array}\right] \xrightarrow[-2R_1+R_3\to R_3]{4R_1+R_2\to R_2} \left[\begin{array}{ccc|c} 1 & -2 & 2 & 0 \\ 0 & -6 & 8+h & 0 \\ 0 & 1 & 0 & 0 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -6 & 8+h & 0 \end{array}\right] \xrightarrow[6R_2+R_3\to R_3]{2R_2+R_1\to R_1} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 8+h & 0 \end{array}\right]$$

Dependent if there is a free variable

$\Rightarrow 8+h = 0$

$\boxed{h = -8}$

(b). TRUE or FALSE: Part (a) is equivalent to asking "For what value(s) of h is $\mathbf{v}_3$ in the Span$\{\mathbf{v}_1, \mathbf{v}_2\}$?"
[No explanation necessary.]

Solve $A\vec{x} = \vec{b}$ where $A = [\vec{v}_1\ \vec{v}_2]$ & $\vec{b} = \vec{v}_3$

i.e. $\left[\begin{array}{cc|c} 1 & -2 & 2 \\ -4 & 2 & h \\ 2 & -3 & 4 \end{array}\right]$ augment.

4. (5 pts) Prove: If a set $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\}$ in $\mathbb{R}^n$ contains the zero vector, then the set is linearly dependent.

Let S be defined as above and let $\vec{v}_k = \vec{0}$.

Then the equation

$0\vec{v}_1 + 0\vec{v}_2 + \ldots + 0\vec{v}_{k-1} + 1\cdot\vec{v}_k + 0\,\vec{v}_{k+1} + \cdots + 0\vec{v}_p = \vec{0}$.

holds for weights not all zero since $c_k = 1$ ($\&\ \vec{v}_k = \vec{0}$)

$\therefore$ S is linearly dependent. ∎