Name: Key

Math 362 Linear Algebra – Crawford

Quiz 1

13 September 2017

Books, calculators, and notes (in any form) are not are allowed. Show all your work for credit. ***Good luck!***
[Note: Each quiz score will be scaled to 15 points after grading.]

1. (8 pts) Given $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$, $\mathbf{a}_3 = \begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix}$.

(a). Determine whether $\mathbf{b} = \begin{bmatrix} -2 \\ 5 \\ 4 \end{bmatrix}$ is a linear combination of $\mathbf{a}_1, \mathbf{a}_2$ and $\mathbf{a}_3$.

If it is, find the weights and write $\mathbf{b}$ as the linear combination found.

$$\begin{bmatrix} 1 & 0 & -3 & -2 \\ 3 & 1 & -2 & 5 \\ 2 & 2 & 1 & 4 \end{bmatrix} \xrightarrow[-2R_1+R_3 \to R_3]{-3R_1+R_2 \to R_2} \begin{bmatrix} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 2 & 7 & 8 \end{bmatrix}$$

$$\xrightarrow{-2R_2+R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 0 & -7 & -14 \end{array}\right] \xrightarrow{-\frac{1}{7}R_3 \to R_3} \begin{bmatrix} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

note: At this stage (REF) we know that there is a sol^n ie $\vec{b}$ can be written as a lin. comb. Keep going (RREF) to find weights.

$$\xrightarrow[-7R_3+R_2 \to R_2]{3R_3+R_1 \to R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

ie $x_1 = 4$, $x_2 = -3$, $x_3 = 2$

ie $\boxed{\vec{b} = 4\vec{a}_1 - 3\vec{a}_2 + 2\vec{a}_3}$

[Don't forget to write $\mathbf{b}$ as a linear combination found, if found.]

(b). Do $\mathbf{a}_1, \mathbf{a}_2$ and $\mathbf{a}_3$ span $\mathbb{R}^3$? Pivot in every row $\Rightarrow$ $\boxed{\text{Yes}}$

2. (2 pts) Write a matrix equation that is equivalent to the following vector equation.

$$c_1 \begin{bmatrix} -2 \\ 5 \end{bmatrix} + c_2 \begin{bmatrix} -3 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} -2 & -3 & 1 \\ 5 & 0 & 2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}}$$

2x3 3x1 2x1 ✓

3. (10 pts)

(a). Given $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, show that $\begin{bmatrix} h \\ k \end{bmatrix}$ is in Span $\{\mathbf{u}, \mathbf{v}\}$ for all h and k. [Briefly explain how you know.]

$$\begin{bmatrix} 1 & 3 & h \\ 2 & 4 & k \end{bmatrix} \xrightarrow{-2R_1+R_2 \to R_2} \left[\begin{array}{cc|c} 1 & 3 & h \\ 0 & -2 & -2h+k \end{array}\right]$$

Pivot in each row of $[\vec{u}\ \vec{v}]$ matrix, so it is not possible to get a row $[0\ 0 : \blacksquare]$ ← not zero. ∴ All $\begin{bmatrix} h \\ k \end{bmatrix}$ are in the span$\{\vec{u}, \vec{v}\}$

(b). Given $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$, determine a relationship between h and k such that $\begin{bmatrix} h \\ k \end{bmatrix}$ is in Span $\{\mathbf{u}, \mathbf{v}\}$.

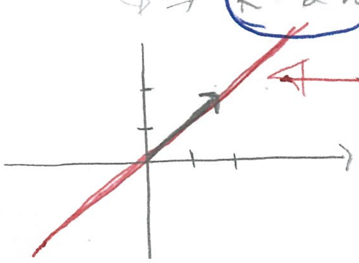
$$\begin{bmatrix} 1 & 2 & h \\ 2 & 4 & k \end{bmatrix} \longrightarrow \left[\begin{array}{cc|c} 1 & 2 & h \\ 0 & 0 & -2h+k \end{array}\right]$$

Consistent only if $\boxed{-2h+k=0}$

(c). Give a geometric description of and/or sketch the set of all vectors $\begin{bmatrix} h \\ k \end{bmatrix}$ that satisfy the relationship found ~~it~~ in part (b). [Hint: Use the relationship from (b) to rewrite the vector $\begin{bmatrix} h \\ k \end{bmatrix}$.]

$-2h+k=0$

$\Rightarrow k=2h \Rightarrow \begin{bmatrix} h \\ k \end{bmatrix} = \begin{bmatrix} h \\ 2h \end{bmatrix} = h \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ ← $\boxed{\text{line through the origin and } \begin{bmatrix} 1 \\ 2 \end{bmatrix}.}$



← Span$\{\vec{u}, \vec{v}\}$