Name: ______________________________ **Exam 2**

Math 362 Linear Algebra – Crawford **15 November 2017**

Books and notes (in any form) are <u>not</u> are allowed. You may use a calculator – but please indicate when you use the matrix functions on the calculator. Put all of your work and answers on the separate paper provided and staple this cover sheet on top. Show all your work for credit. ***Good luck!*** Calculator # ____________

1. (16 pts) Given the system $\begin{array}{rcrcr} 3sx_1 & - & 5x_2 & = & 3 \\ 9x_1 & + & 5sx_2 & = & 2 \end{array}$, which contains the parameter s,

(a). Use Cramer's Rule to find the solution.

(b). Determine the value(s) of s for which the system has a unique solution.

2. (16 pts) Given $A = \begin{bmatrix} 4 & 4 & -2 & 6 & 0 \\ 1 & 1 & 2 & 4 & 0 \end{bmatrix}$,

(a). Find a basis for Col A.

(b). Find a basis for Nul A.

3. (16 pts) Let H be the set of all matrices in $M_{n\times n}$ such that $A^2 = A$. That is, $H = \{A \text{ in } M_{n\times n} \mid A^2 = A\}$. Determine whether H is a subspace of $M_{n\times n}$. If it is not a subspace, clearly show <u>**all**</u> *subspace* properties that do not hold.

4. (8 pts) Let $\mathbf{v}_1 = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} 7 \\ 9 \\ 3 \end{bmatrix}$. Let $H = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

It can be verified that $2\mathbf{v}_1 + 5\mathbf{v}_2 - 2\mathbf{v}_3 = \mathbf{0}$. Use this information to find a basis for H. [Justify your answer.]

5. (9 pts) Determine whether the following statements are true or false. If the statement is false, correct the statement and/or clearly explain why it is false. [You may answer these questions on this cover sheet, if you would like.]

(a). For an $n \times n$ matrix A, $\det(kA) = k^n \det(A)$.

(b). For an $n \times n$ matrix A, if the $\det(A) = 0$, then two rows or two columns are the same, or a row or a column is zero.

(c). For $A = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \end{bmatrix}$, the determinant is given by $|A| = a\begin{vmatrix} f & g & h \\ j & k & l \end{vmatrix} - e\begin{vmatrix} b & c & d \\ j & k & l \end{vmatrix} + i\begin{vmatrix} b & c & d \\ f & g & h \end{vmatrix}$.

6. (8 pts) Answer the following questions and ***briefly*** explain your answer. [You may answer these questions on this cover sheet, if you would like.]

(a). If A is a 5×5 matrix and the equation $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b}$ in $\mathbb{R}^5$, is it possible that for some $\mathbf{b}$, the equation $A\mathbf{x} = \mathbf{b}$ has more than one solution? Why or why not?

(b). Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation with the property that $T(\mathbf{u}) = T(\mathbf{v})$ for some distinct pair of vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$. Can T map $\mathbb{R}^n$ onto $\mathbb{R}^n$? Why or why not?

7. (30 pts) <u>Prove 3</u> of the following. Clearly state theorems and properties that you use.

<u>Bonus</u>: You may do (or attempt) all four options and each will be graded out of 10 points. Whichever three you score higher on will be your base grade. Any points from the fourth problem will be cut in third and added to your base grade.

(a). Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation and let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a linearly dependent set in $\mathbb{R}^n$. Prove that the set $\{T(\mathbf{v}_1), T(\mathbf{v}_2), T(\mathbf{v}_3)\}$ is also linearly dependent.

(b). (New-ish) If the columns of an $n \times n$ matrix A are linearly independent, prove that the columns of A^2 span $\mathbb{R}^n$.

(c). (New) Let B be an $m \times m$ invertible matrix and let A be an $m \times n$ matrix. If $\mathbf{x}$ is in Nul (BA), prove that $\mathbf{x}$ is in Nul (A).

(d). (New) Let A be an invertible $n \times n$ matrix. Prove that $\det(\operatorname{adj} A) = (\det A)^{n-1}$. [Do ***not*** try to write out the adjoint for an arbitrary $n \times n$ matrix.]