Name: ______________________________ **Exam 1**

Math 362 Linear Algebra – Crawford **04 October 2017**

Books and notes (in any form) are <u>not</u> are allowed. You may use a calculator – but please indicate when you use the matrix functions on the calculator. Put all of your work and answers on the separate paper provided and staple this cover sheet on top. Show all your work for credit. ***Good luck!***

Calculator # __________

1. (12 pts) Given the matrices and vectors, $A = \begin{bmatrix} 1 & -1 \\ -2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -3 \\ 1 & 2 \\ 2 & 0 \end{bmatrix}$, and $\mathbf{x} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$

compute each of the following or explain why it is undefined.

(a). AB^T

(b). $\mathbf{x}\mathbf{x}^T + \mathbf{x}^T\mathbf{x}$

2. (12 pts) Given the linear system

$$\begin{array}{rcrcrcrcl} x_1 & + & 2x_2 & & & + & x_4 & = & 4 \\ & & & & x_3 & - & 3x_4 & = & 2 \\ x_1 & + & 2x_2 & + & 2x_3 & - & 5x_4 & = & 8 \end{array}$$

Find the solution and write it in <u>parametric vector</u> form. *Clearly indicate which part of your solution is the particular solution and which part is the solution to the associated homogeneous equation.*

3. (12 pts) Given that A and B are invertible,

(a). Solve for X in the following equation: $AXB = (AB)^2$ Simplify your answer.

(b). Give a counterexample using 2×2 matrices to show that $(A + B)^{-1} \neq A^{-1} + B^{-1}$.

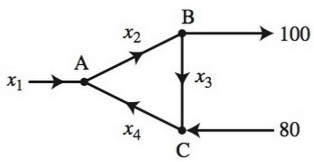
4. (12 pts) Find the inverse of $A = \begin{bmatrix} 1 & x & x \\ 0 & 1 & x \\ 0 & 0 & 1 \end{bmatrix}$.

5. (12 pts) Given the matrix $A = \begin{bmatrix} 2 & 5 & -1 \\ 0 & 1 & -1 \\ 1 & 2 & 0 \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} 4 \\ -1 \\ 4 \end{bmatrix}$, [Justify your answers.]

(a). Is $\mathbf{u}$ in the set spanned by the columns of A?

(b). Are the columns of A linearly independent?

6. (10 pts) Given the network shown in the figure below, ***set up*** the system of equations that describe the general flow pattern, ***but do not solve***.



7. (20 pts) Prove ***two*** of the following.

BONUS: You may do (or attempt) all three options and each will be graded out of 10 points. Whichever two you score the highest on will be your base grade. Any points from the other problem will be cut in third and added to your base grade. (e.g. If you get 10 points, 5 points, and 3 points, then your score will be $10 + 5\frac{3}{3} = 16$)

(a). (NEW-ISH) If $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ are in $\mathbb{R}^n$ and $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent, then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is also linearly dependent.

(b). (NEW) If A and B are invertible and $AB = BA$, then $(AB)^{-1} = A^{-1}B^{-1}$.

(c). (NOT NEW) Suppose $\mathbf{w}$ is a solution of $A\mathbf{x} = \mathbf{b}$ and $\mathbf{u}_h$ is a solution to the homogeneous equation $A\mathbf{x} = \mathbf{0}$. Prove that $\mathbf{v} = \mathbf{w} + \mathbf{u}_h$ is a solution to $A\mathbf{x} = \mathbf{b}$.

8. (12 pts) Determine whether the following statements are true or false. If the statement is false, correct the statement and/or clearly explain why it is false.

[Assume the sizes of any matrices or vectors are such that the sums and products are defined.]

(a). $(A + B)^2 = A^2 + 2AB + B^2$.

(b). If an $m \times n$ matrix A has a pivot position in every column, then for each $\mathbf{b}$ in $\mathbb{R}^m$, the equation $A\mathbf{x} = \mathbf{b}$ has a solution.

(c). The span of two vectors in $\mathbb{R}^3$ can never span all of $\mathbb{R}^3$.

(d). Matrix multiplication of two 3×3 matrices can be defined as $AB = [A\mathbf{b}_1 \ A\mathbf{b}_2 \ A\mathbf{b}_3]$, where $\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3$ are the columns of B.