Estimating Population Proportions

1. Suppose that in 1996, 867 people who work in the Chicago suburbs located in Lake, Cook, and DuPage Counties were surveyed about commuting habits. 635 responded that they drove their car alone to work (rather than carpool, train, bike, walk, etc.).

- (a). What is the sample? How big is the sample?
- (c). What is the population? Can you determine how big it is?
- (b). What is the exact proportion of the sample who drove alone?
- (d). Do you know exact proportion of the population who drove alone?

If not, what value you would you use as an estimate?

[STOP HERE UNTIL READY TO CONTINUE.]

Notation/Terminology:

p =

 $\hat{p} =$

 $\hat{q} =$

What is the difference between a POINT ESTIMATE and an INTERVAL ESTIMATE for a population parameter?

We can use the Normal Distribution to approximate the Binomial Distribution (for the sample proportion) when the following conditions are satisfied:

- (a). Sample is a simple random sample.
- (b). The conditions for the binomial distribution are satisfied.

- (c). There are at least 5 successes and 5 failures.
- Consider the Standard Normal Distribution:

2. Find the critical value $z_{\alpha/2}$ for the following confidence levels.

(a). 99%

(b). 95%

(c). 90%

We will use the <u>CONFIDENCE LEVEL</u> (or DEGREE OF CONFIDENCE) and the <u>CRITICAL VALUE</u> $z_{\alpha/2}$ to get a <u>CONFIDENCE INTERVAL</u> (i.e. interval estimate) for the population parameter p.

But we need one more value first:

Steps to find the Confidence Interval (CI):

- 1. Verify the that requirements to use the Normal Distribution as an approximation to the Binomial Distribution are satisfied.
- 2. Given a <u>Confidence Level</u>, use Table A2 to find the <u>associated critical value</u> $z_{\alpha/2}$
- **3.** Compute the Margin or Error $E = z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$
 4. The Confidence Interval for the population proportion p is given by

 $\hat{p} E
 Alternate Forms:
 <math>\hat{p} \pm E$ $(\hat{p} E, \hat{p} + E)$

 $\underline{\mathbf{Ex}}$ (a). Find the 95% confidence interval for the commuter example.

(b). Interpret the results.

<u>Ex</u> Given the confidence interval .256 < p < .413, find the point estimate \hat{p} and the margin of error E.

Suppose we want to collect *sample* data to estimate a *population* proportion.

How big should the sample size be (given a desired confidence level and margin or error)?

• If estimate \hat{p} is known, use $n = \frac{\left(z_{\alpha/2}\right)^2 \cdot \hat{p}\hat{q}}{E^2}$

• If estimate
$$\hat{p}$$
 is not known, use $n = \frac{(z_{\alpha/2})^2 \cdot (.25)}{E^2}$

Ex Suppose we want to estimate the current proportion of commuters who drive alone. How many people must be surveyed in order to be 90% confident that the sample proportion has an error no more than 3 percentage points?

(a). Assume the data from the 1996 survey gives a reasonable approximation of \hat{p} .

(b). With rising costs, more people are finding commuting alternatives. Assume the 1996 data is no longer valid.