

OTHER TYPES OF INDETERMINATE FORMS

1. QUOTIENTS: So far we have primarily looked at INDETERMINATE FORMS that are quotients: $\frac{0}{0}, \frac{\pm\infty}{\pm\infty}$. Hence, the following quotients are DETERMINATE FORMS, *meaning you know the limit*. Indicate the following limits, for a constant $c \neq 0$ [Note: You may indicate $\pm\infty$]:

(a). $\frac{c}{0} \rightarrow \pm\infty$ (b). $\frac{0}{c} \rightarrow 0$ (c). $\frac{\pm\infty}{c} \rightarrow \pm\infty$ (d). $\frac{c}{\pm\infty} \rightarrow 0$

2. PRODUCTS. Only one of the following products gives an indeterminate form. Circle this one and then state the value of the limit for the other four forms.

$0 \cdot \pm\infty$ **IND** $c \cdot 0 = 0$ $c \cdot \pm\infty = \pm\infty$ $\infty \cdot \infty = \infty$ $-\infty \cdot \infty = -\infty$

3. SUMS AND DIFFERENCES. Only one of the following gives an indeterminate form. Circle this one and then state the value of the limit for the other two forms.

$-\infty - \infty = -\infty$ $\infty - \infty$ **IND** $\infty + \infty = \infty$

4. POWERS. Fill in the following blanks. The following two powers are DETERMINATE FORMS:

$$0^\infty$$

$$0^{-\infty}$$

(a). Zero multiplied by itself over and over is still zero. So in the limit 0^∞ , we get 0 since it doesn't matter whether you approach zero from above or below, it still goes to zero.

(b). $0^{-\infty} = \frac{1}{0^\infty} = \frac{1}{0} \rightarrow \pm\infty$

The following three powers are INDETERMINATE FORMS:

$$0^0$$

$$\infty^0$$

$$1^\infty$$

(a). Zero raised to any number should still be zero. But any number raised to the zero should be one. So 0^0 is INDETERMINATE because "reason" gives two plausible answers.

(b). Infinity (or a really big number) raised to a power should still be a really big number, but any number raised to the zero should be one. So ∞^0 is INDETERMINATE because "reason" gives two plausible answers.

(c). 1.00001 multiplied by itself over and over will get larger. But 0.99999 multiplied by itself over and over will get smaller. So the limit 1^∞ is also an INDETERMINATE form.

5. For each of the following limits, clearly state which indeterminate form is obtained. Then do “MORE WORK” (e.g., factor and cancel, limit at infinity shortcuts, etc.) to determine the limit.

(a). $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$

(b). $\lim_{x \rightarrow \infty} \frac{5x^2 - 1}{2x^2 - 4}$

6. Answer the following questions about each limit.

(a). Observe that $\lim_{x \rightarrow \infty} \frac{\ln x}{x - 1}$ results in the Indeterminate Form $\frac{\infty}{\infty}$

Do you think the top is going to infinity

FASTER, SLOWER, or AT THE SAME RATE as the bottom? [Circle One]

Based on your answer do you think the limit will be

∞ 0 or A FINITE NONZERO NUMBER? [Circle One]

Graph the function and see if you were correct about the limit.

(b). Observe that $\lim_{x \rightarrow \infty} \frac{1 - e^{2x}}{x^2}$ results in the Indeterminate Form $\frac{-\infty}{\infty}$

Do you think the top is going to negative infinity

FASTER, SLOWER, or AT THE SAME RATE as the bottom? [Circle One]

Based on your answer do you think the limit will be

$-\infty$ 0 or A FINITE NONZERO NUMBER? [Circle One]

Graph the function and see if you were correct about the limit.