

We now have a “bag of tools” for integrations: **direct, u -substitution, integration by parts, trigonometric integrals, trig substitution, etc.**

But there are still lots of integrals for which we still have no method.

eg. $\int e^{x^2} dx$

$$\int_{-1}^1 \sqrt{1+x^3} dx$$

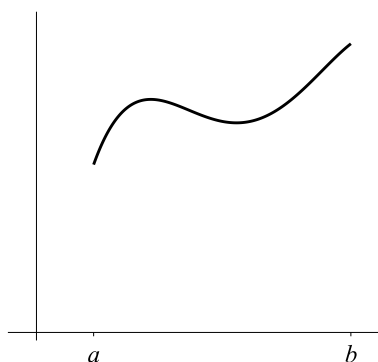
$$d = \int_0^3 v(t) \quad \text{where}$$

t	$v(t)$
0	0
1	10.8
2	15.7
3	29.6

Recall, the Definition of a Definite Integral

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \cdot \Delta x$$

Take the limit off and we have an approximation using n rectangles.



3 Choices for the Rectangles:

- Left Endpoint: $\int_a^b f(x) dx \approx L_n = \sum_{i=0}^{n-1} f(x_i) \cdot \Delta x = \Delta x \cdot [f(x_0) + f(x_1) + \cdots + f(x_{n-1})]$

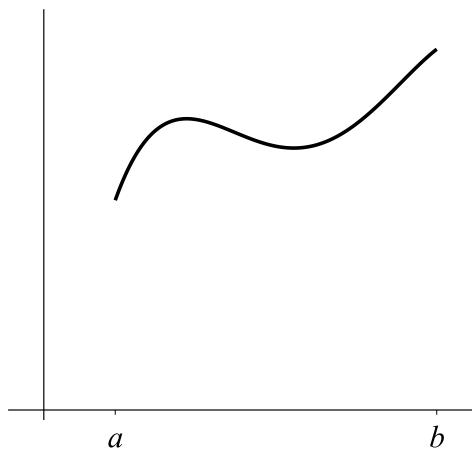
- Right Endpoint: $\int_a^b f(x) dx \approx R_n = \sum_{i=1}^n f(x_i) \cdot \Delta x = \Delta x \cdot [f(x_1) + f(x_2) + \cdots + f(x_n)]$

- Midpoint: $\int_a^b f(x) dx \approx M_n = \sum_{i=1}^n f(\bar{x}_i) \cdot \Delta x = \Delta x \cdot [f(\bar{x}_1) + f(\bar{x}_2) + \cdots + f(\bar{x}_n)]$

where $\bar{x}_i$ is the midpoint of the i^{th} subinterval. i.e. $\bar{x}_i = \frac{x_{i-1} + x_i}{2}$

But approximations with rectangles have errors because **flat tops are used to approximate a changing curve**.

So to reduce error, use slanted tops $\Rightarrow$ Each section approximated with a trapezoid



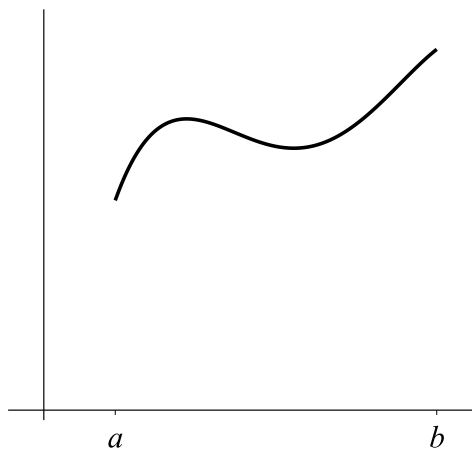
Trapezoid

Rule:

$$\int_a^b f(x) dx \approx T_n = \frac{1}{2} \Delta x [f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

Although error typically reduced, still using straight lines to approximate curved tops

So to reduce error, use (curved) parabolas to approximate the top of each section



Simpson's

Rule:

$$\int_a^b f(x) dx \approx S_n = \frac{1}{3} \Delta x [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$