<u>Ex</u> Suppose $T = f(x)$ represents the temperature in a rod a position x. Suppose we take a temperature measurement and want to know the position based on that temperature.

ie. x is now a function of T . Mathematically: $x = g(T)$.

Draw Picture

<u>Def</u> A function g is the <u>INVERSE FUNCTION</u> of the function f if

- $f(g(x)) = x$ for all x in dom(g)
- $g(f(x)) = x$ for all x in dom(f)

Notation: The inverse function is denoted by f^{-1} Important: $f^{-1} \neq \frac{1}{f}$

So to show that 2 functions are inverses of each other, you must show that the definition is satisfied, i.e.:

(1). Both cancelation equations are satisfied:

$f(f^{-1}(x)) = x \qquad \forall x \in \text{dom}(f^{-1})$

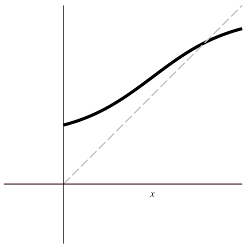
$f^{-1}(f(x)) = x \qquad \forall x \in \text{dom}(f)$

(2). The domain and range must interchange:
ie.

<u>Ex</u> Verify the $f(x) = \frac{1}{\sqrt{x-2}}$ has the inverse $f^{-1} = \frac{1}{x^2} + 2$.
(And find the domain and range of both.)

Since the domain and range interchange $\Longrightarrow$ If the point (a, b) is on f, then the point (b,a) is on f^{-1}.

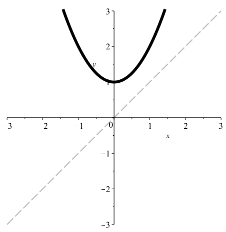
<u>Ex</u> Use this fact to sketch the inverse of $f(x)$. f^{-1} is the reflection of f through the line $y = x$.



$\Rightarrow$ Use the line $y = x$ as a mirror.

Will a function f always have an inverse function? If not, how can we tell?

<u>Ex</u> Given the graph of $f(x) = x^2 + 1$ below, sketch its reflection through the line $y = x$.
Is the reflection a function? No. It fails the VLT and cannot be a *function* $\Rightarrow$ An inverse *function* does not exist.

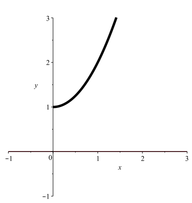


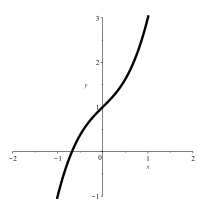
Since the reflection must past the VLT, the original function must past the HLT.

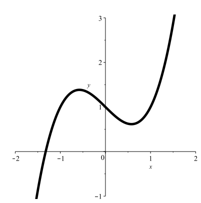
<u>Def</u> A function is called <u>ONE-TO-ONE</u> if it never takes on the same y-value more than once.
ie. $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$.

- A function is 1-1 if and only if it passes the Horizontal Line Test
- A function has an inverse function if and only if it is 1-1

<u>Ex</u> Determine whether the following functions will have an inverse.







<u>Steps for finding and inverse of $f(x)$</u>

0. Verify that an inverse exists.

1. Write $y = f(x)$.

2. Solve for x in terms of y (if possible).

3. Interchange x and y and write $y = f^{-1}(x)$.

4. Define $\text{dom}(f^{-1})$ as the range of f.

<u>Ex</u> Find the inverse function of $f(x) = \sqrt{2x - 3}$