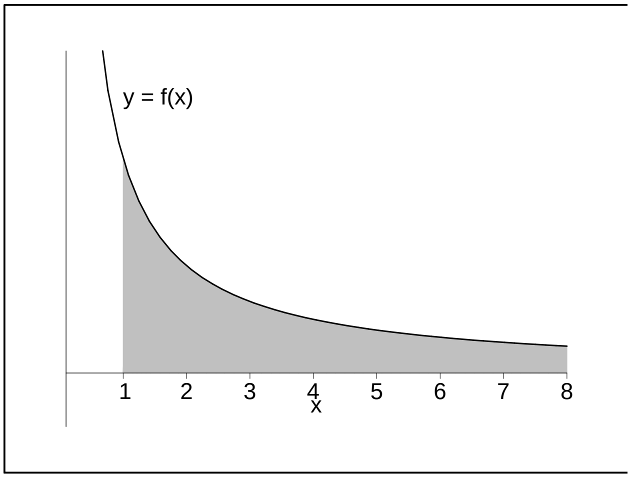
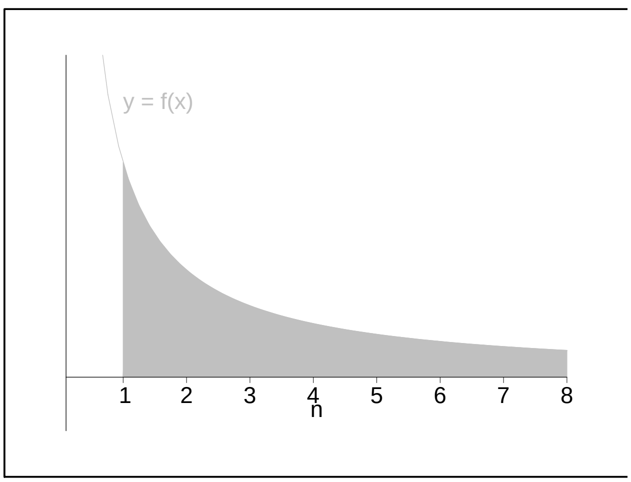
1. Given a continuous, decreasing function $f(x) > 0$, write down an (improper) integral that represents the area under this curve for $x \geq 1$. [Note: $f(x)$ is an arbitrary function in the graphs below.]



2. Let a_n be a sequence where $f(n) = a_n$. Plot and *label* the points $a_1, a_2, a_3, \ldots$ on the graph below.



3. Estimate the area under the curve using the right endpoint of each subinterval to form rectangles. Sketch and shade these rectangles on the graph.
Is this area bigger or smaller than the original shaded area under the curve?

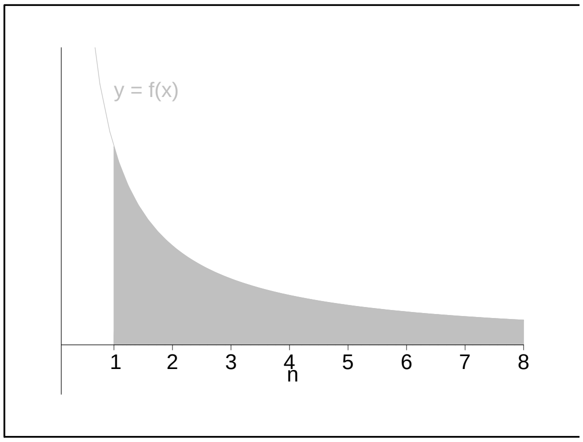
(a). What is the width Δx each rectangle?
What is the height (in terms of the a_n's) of the $1^{st}, 2^{nd}, 3^{rd}, 4^{th}, \ldots$ rectangle?

(b). What is the area of the $1^{st}, 2^{nd}, 3^{rd}, 4^{th}, \ldots$ rectangle?

(c). What is the total area of all these (infinitely many) rectangles?

(d). Write this total area in summation form (i.e. as a series including the correct bounds).

4. Repeat problem #3 using the left endpoints to form rectangles.



Estimate the area under the curve using the left endpoint of each subinterval to form rectangles. Sketch and shade these rectangles on the graph.
Is this area bigger or smaller than the original shaded area under the curve?

(a). What is the width Δx each rectangle?
What is the height (in terms of the a_n's) of the $1^{st}, 2^{nd}, 3^{rd}, 4^{th}, \ldots$ rectangle?

(b). What is the area of the $1^{st}, 2^{nd}, 3^{rd}, 4^{th}, \ldots$ rectangle?

(c). What is the total area of all these (infinitely many) rectangles?

(d). Write this total area in summation form (i.e. as a series including the correct bounds).

5. In the space below write the expressions you obtained for the 3 areas (#1, 3d, 4d):

(1). Area under the curve:

(3d). Area using right endpoints:

(4d). Area using left endpoints:

Put those areas in order of smallest to largest in the inequality below.

$$0 \leq \qquad \leq \qquad \leq$$

Use the results of #2-4 this inequality to answer the questions 6 and 7.

6. Suppose the improper integral $\int_1^\infty f(x)\,dx$ converges [i.e it evaluates to a number].

(a). Does $\sum_{n=2}^{\infty} a_n$ converge or diverge? Why?

(b). If $\sum_{n=2}^{\infty} a_n$ converges, does $\sum_{n=1}^{\infty} a_n$ converge? Why?

7. Suppose the improper integral $\int_1^\infty f(x)\,dx$ diverges (to $+\infty$).

(a). Does $\sum_{n=1}^{\infty} a_n$ converge or diverge? Why?

(b). If $\sum_{n=1}^{\infty} a_n$ diverges, does $\sum_{n=2}^{\infty} a_n$ diverge? Why?

8. Recall from Section 7.8, for which values of p does the integral $\int_1^\infty \frac{1}{x^p}\,dx$ converge?

When does the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converge?

9. Read the statement of The Integral Test on p. 740 of the textbook.

(a). What 3 conditions are required of the function $f(x)$?

(b). Suppose we are interested in a series where n starts at 4. e.g. $\sum_{n=4}^{\infty} \frac{1}{n \ln n}$. Do you think you can still use the integral test? If so, what integral will you use?

$\sum_{n=1}^{\infty} a_n \neq \int_1^{\infty} f(x)\, dx$

Ex: $\sum_{n=1}^{\infty} \frac{3}{n(n+3)} = \frac{11}{6} \approx 1.8333 \neq \int_1^{\infty} \frac{3}{x(x+3)}\, dx = \ln 4 \approx 1.3863$

If $\sum a_n$ converges then $\sum a_n = s$ $\longleftarrow$ Recall, the k^{th} partial sum $s_k = \sum_{n=1}^{k} a_n$

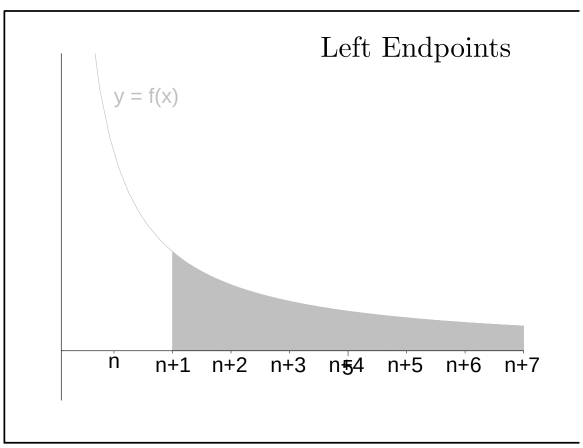
Ex: **(a).** Approximate $s = \sum_{n=1}^{\infty} \frac{1}{n^2}$ using 10 terms:

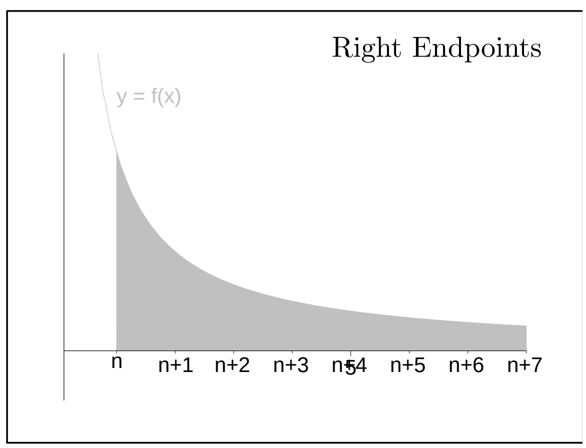
$s \approx s_{10} = \sum_{n=1}^{10} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{100} \approx 1.5498$

But how big is error, if we don't know s?

Define

$$\begin{aligned} R_n &= s - s_n \\ &= (a_1 + a_2 + a_3 + \cdots + a_n + a_{n+1} + a_{n+2} + \cdots) - (a_1 + a_2 + a_3 + \cdots + a_n) \\ &= a_{n+1} + a_{n+2} + a_{n+3} + \cdots \end{aligned}$$





$\int_{n+1}^{\infty} f(x)\, dx \leq$ ____________________ ____________________ $\leq \int_n^{\infty} f(x)\, dx$

$\Rightarrow \int_{n+1}^{\infty} f(x)\, dx \leq$ __________ $\leq \int_n^{\infty} f(x)\, dx$

The largest the error can be is ________________