1. $\displaystyle\sum_{n=1}^{\infty} \frac{-1}{n^2+7n+12}$ Note: $\lim a_n =$ ________ and leading order ________

Rewrite $\dfrac{-1}{n^2+7n+12}$ in partial fraction decomposition (i.e.)

(a). Factor denominator $\dfrac{-1}{n^2+7n+12} = \dfrac{-1}{(n+3)(n+4)}$

(b). "Break up" fraction: $\dfrac{-1}{(n+3)(n+4)} = \dfrac{}{n+3} + \dfrac{}{n+4}$ Need to determine and .

(c). Recombine with ____________________ : $\dfrac{-1}{(n+3)(n+4)} = \dfrac{}{(n+3)(n+4)}$

Numerators must be equal: $-1 =$ $= (A+B)n + (4A+3B)$

Collect Terms: LHS = RHS

n: 0 $= A+B$

constant: -1 $= 4A+3B$

Solve for A and B

(d). Then $\dfrac{-1}{(n+3)(n+4)} = \dfrac{}{n+3} + \dfrac{}{n+4}$

So does $\displaystyle\sum_{n=1}^{\infty} \frac{-1}{n^2+7n+12} = \sum_{n=1}^{\infty}\left(\frac{-1}{n+3} + \frac{1}{n+4}\right)$ converge? If so, to what?

Look at partial sums s_k:

$$
\begin{aligned}
s_1 &= \left(-\tfrac{1}{4}+\tfrac{1}{5}\right) \\
s_2 &= \left(-\tfrac{1}{4}+\tfrac{1}{5}\right)+\left(-\tfrac{1}{5}+\tfrac{1}{6}\right) \\
s_3 &= \left(-\tfrac{1}{4}+\tfrac{1}{5}\right)+\left(-\tfrac{1}{5}+\tfrac{1}{6}\right)+\left(-\tfrac{1}{6}+\tfrac{1}{7}\right) \\
s_4 &= \left(-\tfrac{1}{4}+\tfrac{1}{5}\right)+\left(-\tfrac{1}{5}+\tfrac{1}{6}\right)+\left(-\tfrac{1}{6}+\tfrac{1}{7}\right)+\left(-\tfrac{1}{7}+\tfrac{1}{8}\right) \\
&\vdots \\
s_k &= \left(-\tfrac{1}{4}+\tfrac{1}{5}\right)+\left(-\tfrac{1}{5}+\tfrac{1}{6}\right)+\left(-\tfrac{1}{6}+\tfrac{1}{7}\right)+\left(-\tfrac{1}{7}+\tfrac{1}{8}\right)+\cdots+\left(-\tfrac{1}{k+2}+\tfrac{1}{k+3}\right)+\left(-\tfrac{1}{k+3}+\tfrac{1}{k+4}\right) \\
s_k &=
\end{aligned}
$$

So $\lim s_k =$. Thus the infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{-1}{n^2+7n+12} =$

since ______________________________ .

2. $\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$

$$\frac{3}{n(n+3)} = \frac{A}{n} + \frac{B}{n+3} \Longrightarrow$$

$$\Longrightarrow 3 = (A+B)n + 3A$$

Collect Terms:	LHS	= RHS	
n:		=	$A = 1 \quad B = -1$
constant:		=	

So $\sum_{n=1}^{\infty} \frac{3}{n(n+3)} = \sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+3}$

$$s_1 = \left(1 - \tfrac{1}{4}\right)$$

$$s_2 = \left(1 - \tfrac{1}{4}\right) + \left(\tfrac{1}{2} - \tfrac{1}{5}\right)$$

$$s_3 = \left(1 - \tfrac{1}{4}\right) + \left(\tfrac{1}{2} - \tfrac{1}{5}\right) + \left(\tfrac{1}{3} - \tfrac{1}{6}\right)$$

$$s_4 = \left(1 - \tfrac{1}{4}\right) + \left(\tfrac{1}{2} - \tfrac{1}{5}\right) + \left(\tfrac{1}{3} - \tfrac{1}{6}\right) + \left(\tfrac{1}{4} - \tfrac{1}{7}\right)$$

$$s_5 = \left(1 - \tfrac{1}{4}\right) + \left(\tfrac{1}{2} - \tfrac{1}{5}\right) + \left(\tfrac{1}{3} - \tfrac{1}{6}\right) + \left(\tfrac{1}{4} - \tfrac{1}{7}\right) + \left(\tfrac{1}{5} - \tfrac{1}{8}\right)$$

$$s_6 = \left(1 - \tfrac{1}{4}\right) + \left(\tfrac{1}{2} - \tfrac{1}{5}\right) + \left(\tfrac{1}{3} - \tfrac{1}{6}\right) + \left(\tfrac{1}{4} - \tfrac{1}{7}\right) + \left(\tfrac{1}{5} - \tfrac{1}{8}\right) + \left(\tfrac{1}{6} - \tfrac{1}{9}\right)$$

$$s_7 = \left(1 - \tfrac{1}{4}\right) + \left(\tfrac{1}{2} - \tfrac{1}{5}\right) + \left(\tfrac{1}{3} - \tfrac{1}{6}\right) + \left(\tfrac{1}{4} - \tfrac{1}{7}\right) + \left(\tfrac{1}{5} - \tfrac{1}{8}\right) + \left(\tfrac{1}{6} - \tfrac{1}{9}\right) + \left(\tfrac{1}{7} - \tfrac{1}{10}\right)$$

$$\vdots$$

$$s_k = \left(1 - \tfrac{1}{4}\right) + \left(\tfrac{1}{2} - \tfrac{1}{5}\right) + \left(\tfrac{1}{3} - \tfrac{1}{6}\right) + \left(\tfrac{1}{4} - \tfrac{1}{7}\right) + \left(\tfrac{1}{5} - \tfrac{1}{8}\right) + \left(\tfrac{1}{6} - \tfrac{1}{9}\right) + \left(\tfrac{1}{7} - \tfrac{1}{10}\right) + \cdots$$

$$+ \cdots + \left(\tfrac{1}{k-3} - \tfrac{1}{k}\right) + \left(\tfrac{1}{k-2} - \tfrac{1}{k+1}\right) + \left(\tfrac{1}{k-1} - \tfrac{1}{k+2}\right) + \left(\tfrac{1}{k} - \tfrac{1}{k+3}\right)$$

$$s_k =$$

Then $\lim s_k =$

Therefore the infinite series $\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$ ____________________