Let's expand the idea from the last worksheet to find a power series $\sum_{n=0}^{\infty} c_n x^n$ to represent a generic function $f(x)$

In other words, given $f(x)$ find the coefficients $c_0, c_1, c_2, \ldots$ so that

$$f(x) = \sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + c_5 x^5 + \cdots$$

Start by differentiating the above expression. That is, find the derivatives of $f(x)$ and differentiate the power series. Then evaluate them both at at $x = 0$.

$$f(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + c_5 x^5 + \cdots \qquad f(0) = c_0$$

$$f'(x) = c_1 + 2\,c_2 x + 3\,c_3 x^2 + 4\,c_4 x^3 + 5\,c_5 x^4 + \cdots \qquad f'(0) =$$

$$f''(x) = 2\,c_2 + 2 \cdot 3\,c_3 x + 3 \cdot 4\,c_4 x^2 + 4 \cdot 5\,c_5 x^3 + \cdots \qquad f''(0) =$$

$$f'''(x) = 2 \cdot 3\,c_3 + 2 \cdot 3 \cdot 4\,c_4 x + 3 \cdot 4 \cdot 5\,c_5 x^2 + \cdots \qquad f'''(0) =$$

$$f^{(iv)}(x) = 2 \cdot 3 \cdot 4\,c_4 + 2 \cdot 3 \cdot 4 \cdot 5\,c_5 x + \cdots \qquad f^{(iv)}(0) =$$

$$\vdots$$

$$f^{(n)}(x) = n!\,c_n + (\text{terms multiplied by } x^k) \qquad f^{(n)}(0) =$$

Remember $f(x)$ is given (i.e. known) and hence

$f(0), f'(0), f''(0), \ldots, f^{(n)}(0)$ ___can all be found by taking the derivatives of f___ .

So the only unknown in equation $f^{(n)}(0) = n!c_n$ is ___c_n___ .

Lucky for us, the coefficients c_n are exactly what we set out to find. Solve:

Repeat the process for a power series centered at $x = a$.

In other words, given $f(x)$ find the coefficients $c_0, c_1, c_2, \ldots$ so that

$$f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + c_4(x-a)^4 + c_5(x-a)^5 + \cdots$$

Start by finding the derivatives and evaluating them at $x = a$.

$$f(x) = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + c_4(x-a)^4 + c_5(x-a)^5 + \cdots \qquad f(a) = c_0$$

$$f'(x) = c_1 + 2\,c_2(x-a) + 3\,c_3(x-a)^2 + 4\,c_4(x-a)^3 + 5\,c_5(x-a)^4 + \cdots \qquad f'(a) =$$

$$f''(x) = 2\,c_2 + 2\cdot 3\,c_3(x-a) + 3\cdot 4\,c_4(x-a)^2 + 4\cdot 5\,c_5(x-a)^3 + \cdots \qquad f''(a) =$$

$$f'''(x) = 2\cdot 3\,c_3 + 2\cdot 3\cdot 4\,c_4(x-a) + 3\cdot 4\cdot 5\,c_5(x-a)^2 + \cdots \qquad f'''(a) =$$

$$f^{(iv)}(x) = 2\cdot 3\cdot 4\,c_4 + 2\cdot 3\cdot 4\cdot 5\,c_5(x-a) + \cdots \qquad f^{(iv)}(a) =$$

$$\vdots$$

$$f^{(n)}(x) = n!\,c_n + (\text{terms multiplied by } (x-a)^k) \qquad f^{(n)}(a) =$$

Remember $f(x)$ and a are given (i.e. known) and hence

$f(a), f'(a), f''(a), \ldots, f^{(n)}(a)$ __can all be found by taking the derivatives of f__ .

So the only unknown in equation $f^{(n)}(a) = n!c_n$ is __c_n__ .

Lucky for us, the coefficients c_n are exactly what we set out to find. Solve: