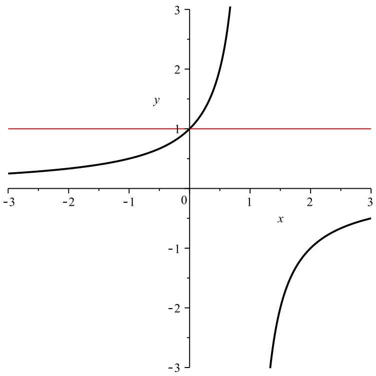
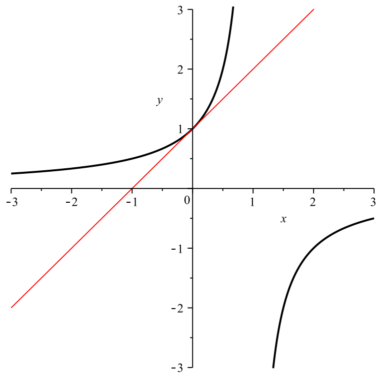
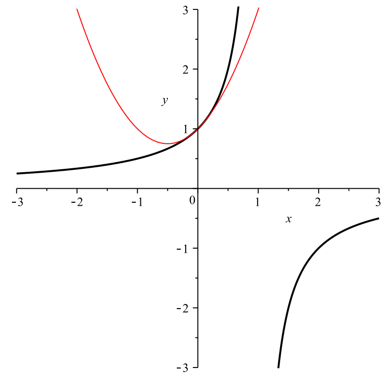
$$\frac{1}{1-x} \approx 1 = p_0(x)$$



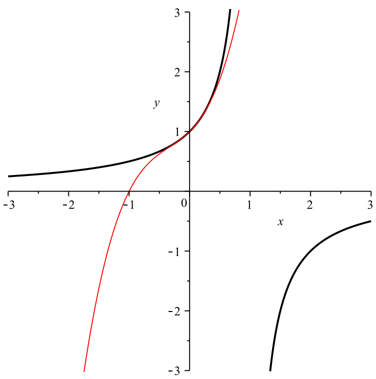
$$\frac{1}{1-x} \approx 1 + x = p_1(x)$$



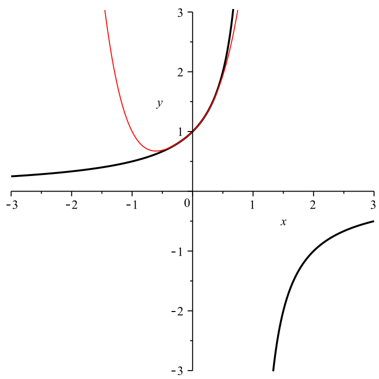
$$\frac{1}{1-x} \approx 1 + x + x^2 = p_2(x)$$



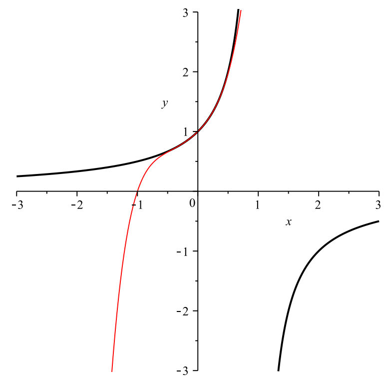
$$\frac{1}{1-x} \approx 1 + x + x^2 + x^3 = p_3(x)$$



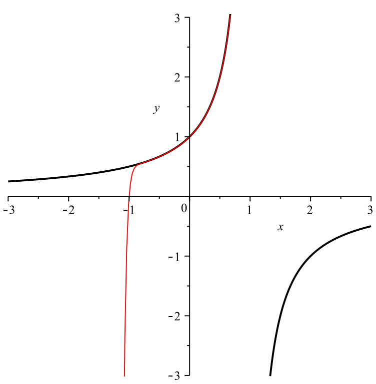
$$\frac{1}{1-x} \approx 1 + x + x^2 + x^3 + x^4 = p_4(x)$$



$$\frac{1}{1-x} \approx 1 + x + x^2 + x^3 + x^4 + x^5 = p_5(x)$$



$$\frac{1}{1-x} \approx 1 + x + x^2 + x^3 + x^4 + x^5 + \ldots + x^{25} = p_{25}(x)$$

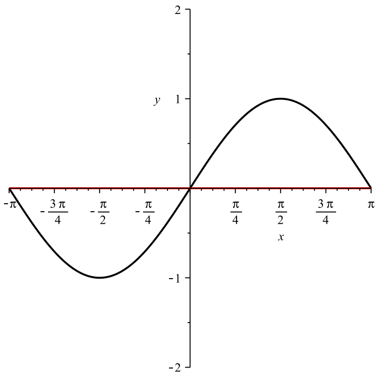


Since the geometric series $\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \ldots$ converges to $\dfrac{1}{1-x}$ for $|x| < 1$,

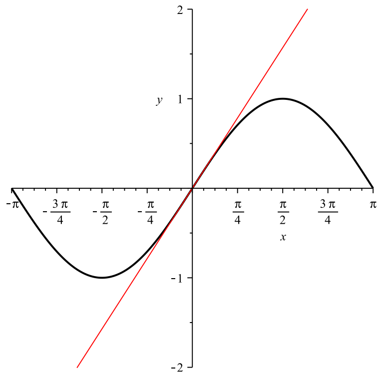
we expect the graphs of $f(x) = \frac{1}{1-x}$ and the n^{th} order approximating polynomial $p_n = 1 + 1 + x + x^2 + x^3 + \ldots x^n$

to match well on the same interval $|x| < 1$.

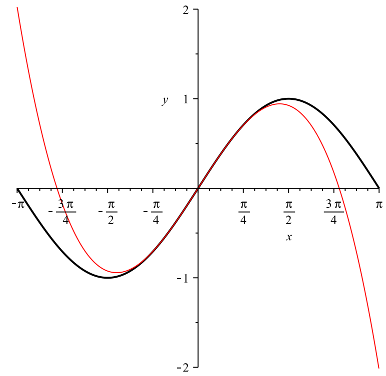
$\sin x \approx 0 = p_0(x)$



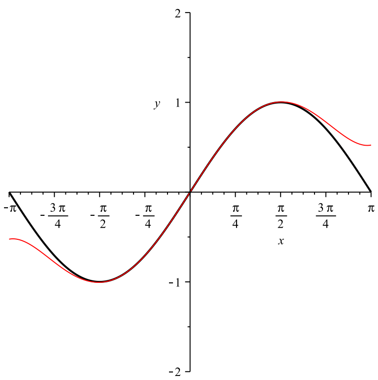
$\sin x \approx x = p_1(x)$



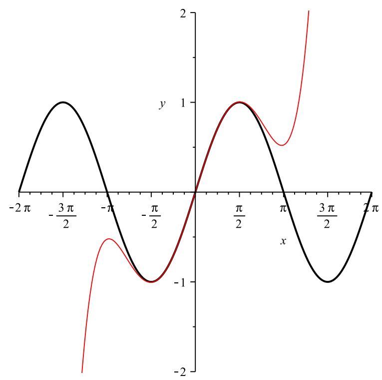
$\sin x \approx x - \frac{1}{6}x^3 = p_3(x)$



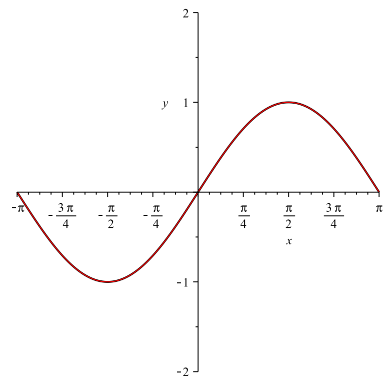
$\sin x \approx x - \frac{1}{6}x^3 + \frac{1}{120}x^5 = p_5(x)$



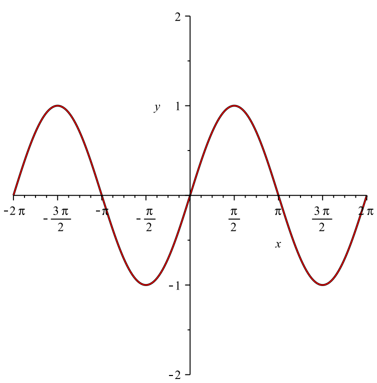
$\sin x \approx x - \frac{1}{6}x^3 + \frac{1}{120}x^5 = p_5(x)$ on $[-2\pi, 2\pi]$



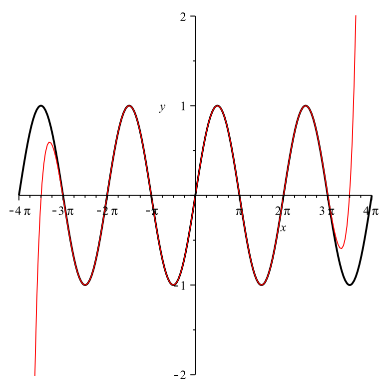
$\sin x \approx x - \frac{1}{6}x^3 + \frac{1}{120}x^5 + \ldots - \frac{1}{25!}x^{25} = p_{25}(x)$



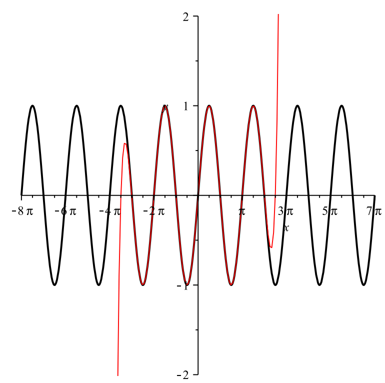
$\sin x \approx p_{25}(x)$on $[-2\pi, 2\pi]$



$\sin x \approx p_{25}(x)$on $[-4\pi, 4\pi]$



$\sin x \approx p_{25}(x)$on $[-8\pi, 8\pi]$



The Ratio test will show that the infinite series $\sum_{n=0}^{\infty} = \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \ldots$ converges for all x. Thus, we expect the graphs of $f(x) = \sin x$ and the n^{th} order approximating polynomial p_n to match well for all x... but we will need more and more terms for it to converge further away from $x = 0$.