

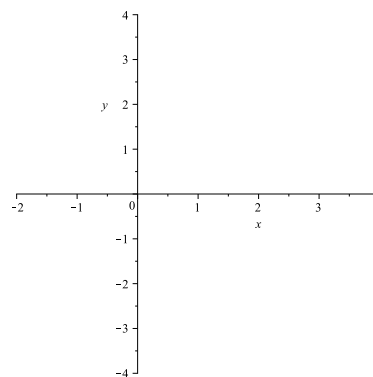
Ex Given $x = \frac{3}{t^2 + 1}$ and $y = t - 1$

Graph the curve on your calculator for $-5 \leq t \leq 5$.

Use a viewing window $-2 \leq x \leq 4$ and $-4 \leq y \leq 4$.

Sketch a copy on the axes to the right.

Then answer the questions below.



(a). Does this curve represent a function? Why or why not?

(b). Could you draw tangent lines to this curve? If so, sketch the tangent line at the point $(\frac{3}{2}, 0)$.

Give an estimate of the slope of the tangent line that you just drew.

(c). Based on part (b), does it make sense to talk about the derivative $\frac{dy}{dx}$ (or slope) of this curve?
Note: This is the derivative of y with respect to x .

So given $x = x(t)$ and $y = y(t)$ how do we find $\frac{dy}{dx}$?

SOLUTION: Suppose we *could* eliminate t and write _____ (*), then $\frac{dy}{dx} =$ _____ (**)

But $y = y(t)$ and $x = x(t)$, so (*) becomes $y(t) =$

Differentiate w/ respect to t : $\frac{d}{dt} [y(t)] = \frac{d}{dt} [F(x(t))]$

Recall, from (**), $\frac{dy}{dx} = F'(x) \Rightarrow$

In Leibniz Notation:

[Note: Formula is still valid even if we can't eliminate t and write $y = F(x)$.]

Ex Given $x = \frac{3}{t^2 + 1}$ and $y = t - 1$

(a). Find $\frac{dy}{dx}$.

(b). Find the slope of the tangent line to the curve at the point $(\frac{3}{2}, 0)$.

(c). Find the equation of the tangent line to the curve at the point $(\frac{3}{2}, 0)$.

(d). Graph this tangent line on your calculator along with the original curve to verify that it is, in fact, the correct tangent line. [Hint: Remember you need to write the tangent line as a parametric curve.]

(e). Find the equation of the tangent line to the curve at the point corresponding to $t = 0$.

Can we find 2^{nd} derivatives? i.e. How do we find $\frac{d^2y}{dx^2}$?

$$\begin{aligned}\frac{d^2y}{dx^2} &= \frac{d}{dx} \left[\frac{dy}{dx} \right] \\ &= \frac{d}{dx} [y'] \\ &= \frac{d(y')}{dx}\end{aligned}$$

but y' is parametrically defined (e.g. function of t), so use the formula from p.1 replacing y with y' .

=

=

=

Warning!! $\frac{d^2y}{dx^2} \neq$

Ex $x = \cos 5t, \quad y = \sin 5t$

(a). Find $\frac{dy}{dx}$.

(b). Find $\frac{d^2y}{dx^2}$.

Ex $x = t^3 - 6t^2, \quad y = t^3 - 12t$

(a). Find the points on the curve where the tangent is horizontal or vertical.

(b). For what values of t is the curve increasing or decreasing?

(c). Use the information obtained in parts (a) and (b) to help sketch the curve on the axes below.

