1. Determine whether the following **series** converge or diverge. Show all your work and clearly indicate any tests that you use.

(a). $\displaystyle\sum_{n=1}^{\infty} \frac{5n^2+n}{3-2n^2}$ Diverges by the Test for Divergence

(b). $\displaystyle\sum_{n=1}^{\infty} \frac{5\sqrt{n}+1}{3+2n^2}$ Converges by Limit Comparison Test

(c). $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$ Diverges by Integral Test

(d). $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2 3^n}{(2n)!}$ Converges by Ratio Test

(e). $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{2n-1}\right)^{3n}$ Converges by the Root Test

(f). $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$ Since $\displaystyle\sum_{n=1}^{\infty} \left|\frac{\cos n}{n^2}\right|$ converges by the Comparison Test (needed positive values), then $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$ converges (absolutely) by Absolute Convergence Test

[More practice can be found in Section 11.7 and the Chapter 11 Review.]

2. Does $\displaystyle\sum_{n=2}^{\infty} (-1)^{n-1}\frac{1}{\ln n}$ converge absolutely, converge conditionally, or diverge. Converges Conditionally since $\displaystyle\sum_{n=2}^{\infty} \left|(-1)^{n-1}\frac{1}{\ln n}\right| = \sum_{n=2}^{\infty} \frac{1}{\ln n}$ diverges (by the Comparison Test), but $\displaystyle\sum_{n=2}^{\infty} (-1)^{n-1}\frac{1}{\ln n}$ converges (by the Alternating Series Test).

3. Given $\displaystyle\sum_{n=0}^{\infty} (-1)^{n}\frac{1}{n+4}$

(a). Find the 5th partial sum s_5. $\displaystyle s_5 = \sum_{n=0}^{5} (-1)^n \frac{1}{n+4} \approx 0.0877$

(b). If s_5 is used to approximate the infinite series, what is the bound for the maximum possible error? (i.e. bound on $|R_5|$?) $|R_5| < b_6 = \dfrac{1}{6+4} = \dfrac{1}{10} = 0.1$

(c). How many terms are needed for error to be less than 0.001. 996 terms

4. How many terms of the series $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n^4}$ are needed to find its sum within 0.01? $n = 5$

5. Find the <u>interval</u> and <u>radius</u> of convergence for the following series.

(a). $\displaystyle\sum_{n=1}^{\infty} \frac{3^n x^n}{n^n}$ $R = \infty$ $(-\infty, \infty)$

(b). $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (x+2)^n}{n}$ $R = 1$ $(-3, -1]$

(c). $\displaystyle\sum_{n=1}^{\infty} \frac{(2x+4)^n}{n4^n}$ $R = 2$ $[-4, 0)$

6. Find the <u>radius</u> of convergence for the following series.

(a). $\displaystyle\sum_{n=0}^{\infty} \frac{(3x-2)^n}{n}$ $R = \frac{1}{3}$

(b). $\displaystyle\sum_{n=0}^{\infty} \frac{n^n x^n}{n!}$ $R = \frac{1}{e}$

7. Use a known power series to find a power series representation for the $f(x) = \dfrac{1}{1+3x^2}$. $\displaystyle = \sum_{n=0}^{\infty} (-1)^n 3^n x^{2n}$

8. Find a Taylor series for $f(x) = \sqrt{x}$ centered at $a = 4$. $\displaystyle f(x) = 2 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1} 1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n-3)}{2^{3n-1} n!} (x-4)^n$

9. Use a known Maclaurin series to find the Maclaurin Series for $f(x) = e^{x/2}$ $\displaystyle e^{x/2} = \sum_{n=0}^{\infty} = \frac{(x/2)^n}{n!} = \sum_{n=0}^{\infty} \frac{x^n}{2^n n!}$

10. Use a known Maclaurin series to evaluate $\displaystyle\int \sin(x^2)\, dx$ as an infinite series. $\displaystyle C + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{x^{4n+3}}{4n+3}$

11. Find the sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{x^{4n}}{n!}$. $\displaystyle\sum_{n=0}^{\infty} \frac{(x^4)^n}{n!} = e^{x^4}$