Name: _____ **Exam 3**

Math 152, Calculus II – Crawford **22 September 2016**

- Calculators, books, or notes (in any form) are not allowed.
- You may use the given formula sheet.
- Clearly indicate your answers.
- ***Show all your work*** – partial credit may be given for written work.
- Good Luck!

	Score
1	/8
2	/30
3	/12
4	/22
5	/10
6	/12
7	/10
Total	/100

1. (8 pts). How many terms are needed to approximate the following infinite series with an error less than 0.01?

$$\sum_{n=1}^{\infty}(-1)^n\frac{4}{n!},$$

Helpful values: $0! = 1$, $1! = 1$, $2! = 2$, $3! = 6$, $4! = 24$, $5! = 120$, $6! = 720$, $7! = 5040$, $8! = 40320$, $9! = 362880$, $10! = 3628800$

2. (30 pts). Determine whether the following series converge or diverge. [Show all your work and clearly indicate any tests that you use.]

(a). $\displaystyle\sum_{n=1}^{\infty}\left(\frac{3n-2n^2}{1+n^2}\right)^{3n}$

(b). $\displaystyle\sum_{n=1}^{\infty}\frac{6n-2}{1+5n^3}$

[Continued on next page. $\longrightarrow$]

(c). $\displaystyle\sum_{n=1}^{\infty} \frac{4+\sin n}{n^2}$

(d). $\displaystyle\sum_{n=1}^{\infty} 6n$

3. (12 pts). Determine whether the following series converges absolutely, converges conditionally, or diverges. [Show all your work and clearly indicate any tests that you use.]

$$\sum_{n=2}^{\infty}(-1)^n\frac{1}{n\ln n}$$

4. (22 pts). Find the radius of convergence and interval of convergence for the following series.

(a). $\sum_{n=0}^{\infty} \frac{4n^2}{n!} x^n$

(b). $\sum_{n=0}^{\infty} \frac{(2x-1)^n}{3n+2}$

5. (10 pts). Use the <u>definition</u> of Taylor Series $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$ to derive the expansion of $f(x) = 3^x$ centered at $\underline{\mathbf{a = 1}}$.

Recall $\frac{d}{dx}[b^x] = b^x \ln b$

[Write your final answer in concise summation notation and simplify. Do **<u>not</u>** find the radius or interval of convergence.]

6. (12 pts). [For this problem, I highly recommend that you use the expanded form of the series.]

(a). Use a known Maclaurin Series to obtain the Maclaurin Series for $f(x) = \dfrac{3x - 3\tan^{-1} x}{x^3}$.
[You must simplify your answer before answering part (b).]

(b). Use the (simplified) series obtained in part (a) to evaluate the following integral $\displaystyle\int \frac{3x - 3\tan^{-1} x}{x^3}\, dx$.
[Show at least 3 nonzero terms in your answer.]

7. (10 pts). TRUE OR FALSE. Determine whether the following statements are true or false.

(a). T F $\displaystyle\sum_{n=0}^{\infty} 4\left(\frac{3}{2}\right) = \frac{4}{1-3/2} = -8.$

(b). T F If $\lim_{k\to\infty} s_k = -3$, then $\displaystyle\sum_{n=1}^{\infty} a_n$ converges, but not necessarily to -3.

Note: $s_k = \displaystyle\sum_{n=1}^{k} a_n$ is the k^{th} partial sum of the infinite series $\displaystyle\sum_{n=1}^{\infty} a_n$.

(c). T F If $\displaystyle\sum_{n=1}^{\infty} c_n x^n$ converges for $x = 3$, but diverges for $x = -5$, then it must converge for $x = -1$.

(d). T F If $\displaystyle\sum_{n=1}^{\infty} c_n x^n$ converges for $x = 3$, but diverges for $x = -5$, then it must converge for $x = -3$.

(e). T F $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1}\frac{3^n}{n} = \ln 4.$