

1. Evaluate the following integrals

$$(a). \int_0^{\pi/2} \sin(9x) \cos(x) \, dx = \frac{1}{10}$$

$$(b). \int \frac{-1}{\sqrt{x^2 - 2x - 3}} \, dx = -\ln \left| \frac{x-1}{2} + \frac{\sqrt{(x-1)^2 - 4}}{2} \right| + C$$

$$(c). \int \tan^3 x \sec^4 x \, dx = \frac{1}{4} \tan^4 x + \frac{1}{6} \tan^6 x + C$$

$$(d). \int \frac{1}{x^3 - 4x^2 + 4x} \, dx = \frac{1}{4} \ln |x| - \frac{1}{4} \ln |x - 2| - \frac{1}{2} \frac{1}{x - 2} + C$$

$$(e). \int x^3 (\ln 2x) \, dx = \frac{1}{4} x^4 \ln(2x) - \frac{1}{16} x^4 + C$$

$$(f). \int \sqrt{9 - x^2} \, dx = \frac{9}{2} \sin^{-1} \left(\frac{x}{3} \right) + \frac{x\sqrt{9 - x^2}}{2} + C$$

$$(g). \int \sin^{-1} x \, dx = x \sin^{-1} x + \sqrt{1 - x^2} + C$$

$$(h). \int \frac{4x}{(x^2 - 1)(x^2 + 1)} \, dx = \ln |x - 1| + \ln |x + 1| - \ln |x^2 + 1| + C$$

2. Use a rationalizing substitution to evaluate $\int \frac{x}{\sqrt{x+9}} dx = \frac{2}{3}(x+9)^{3/2} - 18\sqrt{x+9} + C$ using $u = \sqrt{x+9}$

3. Approximate the integral $\int_4^6 \frac{1}{x^2} dx$, using [Do **NOT** simplify.]

(a). Simpson's Rule with $n = 4$ $\int_4^6 \frac{1}{x^2} dx \approx \frac{1}{6} \left[\frac{1}{16} + 4 \cdot \frac{4}{81} + 2 \cdot \frac{1}{25} + 4 \cdot \frac{4}{121} + \frac{1}{36} \right]$

(b). The Trapezoid Rule with $n = 6$ $\int_4^6 \frac{1}{x^2} dx \approx \frac{1}{6} \left[\frac{1}{16} + 2 \cdot \frac{9}{169} + 2 \cdot \frac{9}{196} + 2 \cdot \frac{1}{25} + 2 \cdot \frac{9}{256} + 2 \cdot \frac{9}{289} + \frac{1}{36} \right]$

4. Evaluate the following improper integral: $\int_0^\infty (7x-3)e^x dx$ Diverges to $+\infty$.

5. Evaluate the following integrals or show that it is a divergent improper integral.

(a). $\int_0^1 \frac{1}{4-2x} dx = \frac{1}{2} \ln 2$ (b). $\int_0^4 \frac{1}{4-2x} dx$ Diverges (c). $\int_{-\infty}^\infty 2x^2 e^{-x^3} dx$ Diverges

6. Integrate: $\int e^x \sin 2x dx = \frac{1}{5} (e^x \sin 2x - 2e^x \cos 2x) + C$

7. Find the area bounded by $f(x) = \sin^2 x \cos^2 x$ on the interval $[0, \pi/2]$. $A = \frac{1}{8}x - \frac{1}{32} \sin 4x \Big|_0^{\pi/2} = \frac{\pi}{16}$

8. Write out the **form** of the partial fraction decomposition for the following function. Do **NOT** determine the values of the coefficients.

$$\frac{3x-4}{x^2(x-2)(x^2+9)^3} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2} + \frac{Dx+E}{x^2+9} + \frac{Fx+G}{(x^2+9)^2} + \frac{Hx+I}{(x^2+9)^3}$$

9. Determine whether the following sequences converge or diverge. Find the limit if it converges.

(a). $a_n = \frac{1}{n} + (-1)^n$ Div./DNE (b). $a_n = \frac{1}{5^n} \rightarrow 0$ (c). $b_n = \frac{\cos n}{n} \rightarrow 0$ (d). $\left\{ \frac{5n^2+n}{3-2n^2} \right\}_{n=1}^\infty \rightarrow -\frac{5}{2}$