1. Given $f(x) = \sin x + 2\cos x$, $-\frac{\pi}{4} \le x \le 0$, find $\left(f^{-1}\right)'(a)$ for $a = 2$. Ans: $\left(f^{-1}\right)'(2) = 1$ Note: $f^{-1}(2) = 0$

2. Solve for x: **(a).** $e^{3x-7} = 6$ $x = \frac{7+\ln 6}{3}$ **(b).** $\ln(x+1) - \ln x = 1$ $x = \frac{1}{e-1}$

3. Given $f(x) = \ln(\cos x + 2)$

(a). What is the domain of f? Domain: all real

(b). Find the relative extreme values on the interval [-1,1]. [Indicate max or min.] Max value of $\ln 3$ at $x = 0$

4. Evaluate the following integrals.

(a). $\displaystyle\int_0^1 \frac{1}{4-2x}\,dx = -\frac{1}{2}\ln|4-2x|\Big|_0^1 = \frac{1}{2}\ln 2$

(b). $\displaystyle\int e^{-7x} - \frac{7\ln x}{x}\,dx = -\frac{1}{7}e^{-7x} - \frac{7}{2}(\ln x)^2 + C$

(c). $\displaystyle\int \frac{3x^2+2x}{x^3+x^2+1}\,dx = \ln|x^3+x^2+1| + C$

(d). $\displaystyle\int \frac{3x}{\sqrt{1-36x^2}}\,dx = -\frac{1}{12}\sqrt{1-36x^2} + C$

(e). $\displaystyle\int \frac{3}{\sqrt{1-36x^2}}\,dx = \frac{1}{2}\sin^{-1}(6x) + C$

(f). $\displaystyle\int \operatorname{sech}^2(5x)\,dx = \frac{1}{5}\tanh(5x) + C$

(g). $\displaystyle\int \sec(5x)\,dx = \frac{1}{5}\ln|\sec(5x) + \tan(5x)| + C$

(h). $\displaystyle\int_0^{\ln 2} \frac{e^{3x}+1}{e^x}\,dx = \frac{1}{2}e^{2x} - e^{-x}\Big|_0^2 = 2$

Simplify using log./exp. properties.

(i). $\displaystyle\int \frac{\tan x}{\ln(\cos x)}\,dx = -\ln|\ln(\cos x)| + C$

5. Given the function $f(x) = e^{-x^2}$

(a). Evaluate $\displaystyle\lim_{x\to-\infty} e^{-x^2} = 0$ **(b).** Evaluate $\displaystyle\lim_{x\to+\infty} e^{-x^2} = 0$ **(c).** Find $f'(x)$ $f'(x) = -2xe^{-x^2}$

6. What is the formula for $\log_a x$ in terms of the natural logarithmic function? $\log_a x = \frac{\ln x}{\ln a}$

7. Find the exact value of **(a).** $\sin(2\tan^{-1}(1)) = 1$ **(b).** $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right) = \frac{5\pi}{6}$

8. Simplify the following expression so that it is an algebraic expression of x: $\cos(\sin^{-1}(2x)) = \sqrt{1-4x^2}$

9. Differentiate the following functions:

(a). $y = x^4 - 4^x + e^{4x} + \ln 4x$
$y' = 4x^3 + 4^x \cdot \ln 4 + 4e^{4x} + \frac{1}{x}$

(b). $y = x^{\frac{1}{x}}$ $\quad y' = \frac{1}{x^2}(1 - \ln x) \cdot x^{1/x}$

(c). $y = \pi^x - \ln e^x$ $\quad y' = \pi^x \cdot \ln \pi - 1$

(d). $h(\theta) = 3\ln\left(\dfrac{2+\cos\theta}{\theta^2}\right)$ $\quad h'(\theta) = \frac{-3\sin\theta}{2+\cos\theta} - \frac{6}{\theta}$

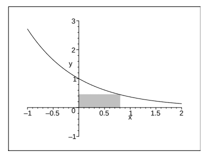
(e). $y = \cosh(\ln(x^3 + 2x^2))$
$y' = \dfrac{3x^2 + 4x}{x^3 + 2x^2}\sinh(\ln(x^3 + 2x^2))$

(f). $f(x) = x\sin^{-1}(3x)$ $\quad f'(x) = \frac{3x}{\sqrt{1-9x^2}} + \sin^{-1}(3x)$

10. Find the equation of the tangent line to $y = \log_3 x$ at $x = 1$ $\quad y = \dfrac{1}{\ln 3}(x-1)$.

11. (a). Find the maximum area of a rectangle in the first quadrant with 2 sides on the x- and y-axes and one vertex on the curve $y = e^{-x}$. See the figure below. [Hint: Express the area of such a rectangle in terms of x only.]
Maximize $A = xy$ subject to $y = e^{-x}$

$\Longrightarrow x = 1$. So max area $= \frac{1}{e}$



(b). Sketch the picture for a rectangle in the first quadrant with 2 sides on the x- and y-axes and one vertex on the curve $y = e^x$. Without using Calculus, determine whether there exists such a rectangle with a maximum area. Briefly explain (a couple of sentences) why or why not. No, if you draw the picture you will see that you can always create a bigger rectangle by taking x further out.

12. Given that a population follows the law of exponential growth, $y(t) = Ce^{kt}$ where y is the population and t is time in years.

(a). Find the proportional constant k, if the population triples every 50 years. $\quad k = \dfrac{\ln 3}{50}$

(b). If the population is 100 after 5 years, find the population at time t. $\quad y = \dfrac{100}{\sqrt[10]{3}}e^{\frac{\ln 3}{50}t} = 89.60e^{.02197t}$

13. Find the following limits. You must clearly show all steps and indicate where you use L'Hopital's Rule.

(a). $\displaystyle\lim_{x\to 0}\frac{\sin 4x}{2x} = 2$ $\quad$ **(b).** $\displaystyle\lim_{x\to\infty}(1+x)^{\frac{1}{x}} = 1$ $\quad$ **(c).** $\displaystyle\lim_{x\to 2}\frac{x-2}{x^2-3x+2} = 1$ $\quad$ **(d).** $\displaystyle\lim_{x\to 2}\frac{x-2}{x^2-3x-2} = 0$