Name: _____ **Exam 1**

Math 152, Calculus II – Crawford **27 September 2016**

- No calculators, books, or notes (in any form) allowed.
- Clearly indicate your answers.
- ***Show all your work*** – partial credit may be given for written work.
- Evaluate trigonometric, exponential, and logarithmic expressions for standard values.
- Good Luck!

Score	
1	/6
2	/12
3	/10
4	/10
5	/12
6	/8
7	/21
8	/14
9	/10
Total	/100

Formulas that you may or may not find helpful

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\sin 2x = 2\sin x \cos x$

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x \, dx = \ln|\csc x - \cot x| + C$$

$$\frac{d}{dx}\left[\sin^{-1} x\right] = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\left[\cos^{-1} x\right] = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left[\tan^{-1} x\right] = \frac{1}{1+x^2} \qquad \frac{d}{dx}\left[\cot^{-1} x\right] = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}\left[\sec^{-1} x\right] = \frac{1}{x\sqrt{x^2-1}} \qquad \frac{d}{dx}\left[\csc^{-1} x\right] = \frac{-1}{x\sqrt{x^2-1}}$$

1. (6 pts). Find the domain of $f(x) = \dfrac{x}{\ln(x-1)}$.

2. (12 pts).

(a). Find the ***exact*** value of $\sin^{-1}\left(\tan\dfrac{3\pi}{4}\right)$.

(b). Use a right triangle to simplify the following expression so that it is an algebraic expression of x.

$$\tan\left(\sin^{-1}\left(\frac{2x}{3}\right)\right)$$

3. (10 pts). Given $f(x) = \sqrt{x^3 + 1}$, find $\left(f^{-1}\right)'(3)$ [Note: f is one-to-one.]

4. (10 pts). Find the equation of the tangent line to $y = \dfrac{e^x}{2x}$ at $x = 1$.

5. (12 pts). Differentiate the following functions. [***Do <u>not</u> simplify.***]

(a). $s(t) = 3^{t^2} + \log_3\left(t^3\right)$

(b). $y = \arcsin\left(\sqrt{\sin x}\right)$

6. (8 pts). Find y' in terms of x only for

$y = x^{\ln x}$

7. (21 pts). Evaluate the following integrals.

(a). $\int \sinh(10x)\, dx$

(b). $\int \dfrac{e^{3x}}{4 + e^{3x}}\, dx$

(c). $\displaystyle\int_0^{\pi/2} \frac{\cos x}{1 + \sin^2 x}\, dx$ Simplify your answer.

8. (14 pts). Evaluate the following limits. Clearly indicate all steps.

(a). $\lim_{x\to 0} \dfrac{3x^2}{1-\cos x}$

(b). $\lim_{x\to 0} (1-3x)^{1/x}$

9. (10 pts). The growth of a bacteria culture follows the model of exponential population growth $P(t) = P_0 e^{kt},\ k > 0$. If the initial bacteria count is 200 and after 4 hours it is 500,

(a). Find the relative growth rate k and write the expression for the number of bacteria after t hours.

(b). When will the bacteria count reach 800? [You do not need to simplify your answers.]