We now have a “bag of tools” for integrations: direct, u-substitution, integration by parts, trigonometric integrals, trig substitution, etc.

But there are still lots of integrals for which we still have no method.

eg. $\int e^{x^2}\,dx$

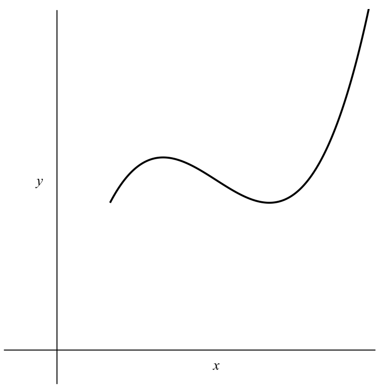
$\int_{-1}^{1} \sqrt{1+x^3}\,dx$

t	$v(t)$
0	0
1	10.8
2	15.7
3	29.6

Recall, the Definition of a Definite Integral

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot \Delta x$$

Take the limit off and we have an approximation using n rectangles.



3 Choices for the Rectangles:

- Left Endpoint: $\int_a^b f(x)\,dx \approx L_n = \sum_{i=0}^{n-1} f(x_i)\cdot\Delta x = \Delta x\cdot[f(x_0)+f(x_1)+\cdots+f(x_{n-1})]$

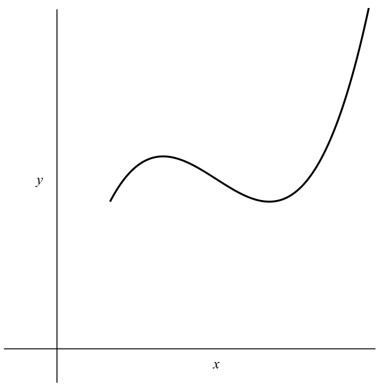
- Right Endpoint: $\int_a^b f(x)\,dx \approx R_n = \sum_{i=1}^{n} f(x_i)\cdot\Delta x = \Delta x\cdot[f(x_1)+f(x_2)+\cdots+f(x_n)]$

- Midpoint: $\int_a^b f(x)\,dx \approx M_n = \sum_{i=1}^{n} f(\bar{x}_i)\cdot\Delta x = \Delta x\cdot[f(\bar{x}_1)+f(\bar{x}_2)+\cdots+f(\bar{x}_n)]$

  where $\bar{x}_i$ is the midpoint of the i^{th} subinterval. i.e. $\bar{x}_i = \dfrac{x_{i-1}+x_i}{2}$

But approximations with rectangles have errors because flat tops are used to approximate a changing curve .

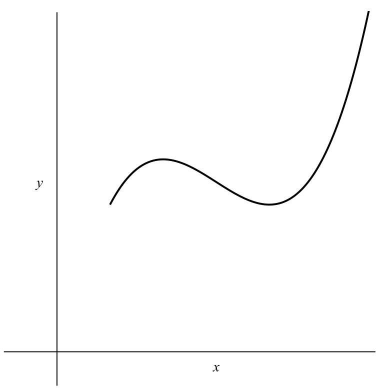
So to reduce error, use slanted tops $\Rightarrow$ Each section approximated with a trapezoid



Trapezoid Rule: $\displaystyle\int_a^b f(x)\,dx \approx T_n = \frac{1}{2}\Delta x\left[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$

Although error typically reduced, still using straight lines to approximate curved tops

So to reduce error, use (curved) parabolas to approximate the top of each section



Simpson's Rule: $\displaystyle\int_a^b f(x)\,dx \approx S_n = \frac{1}{3}\Delta x\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$