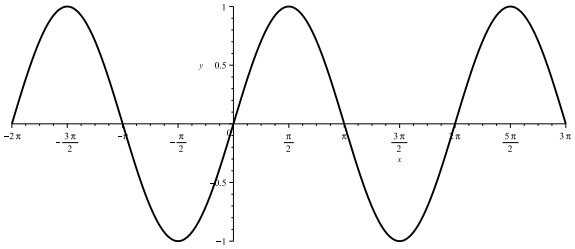
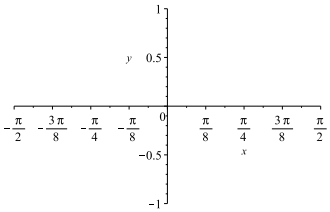
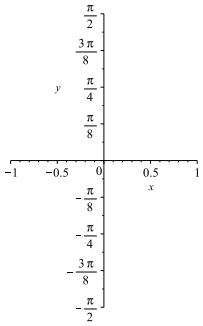
Given the graph of $f(x) = \sin x$, is it one-to-one?







By definition of inverse functions: $\sin^{-1}(x) = y \iff \sin y = x$ for $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

i.e. $\sin^{-1} x$ returns the number between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose sine is x.

Ex $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$ NOT $\frac{4\pi}{3}, \frac{5\pi}{3}$, etc.

Ex $\arcsin\left(\tan\frac{\pi}{4}\right) = \arcsin(1) = \frac{\pi}{2}$

CANCELLATION EQUATIONS

$\sin^{-1}(\sin x) = x$ if $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

$\sin(\sin^{-1} x) = x$ if $-1 \le x \le 1$

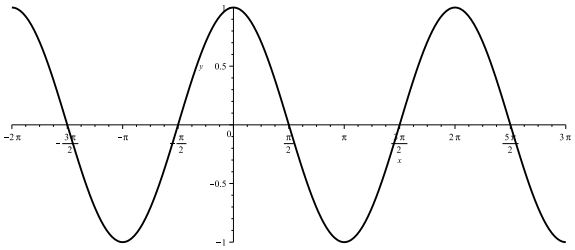
Ex $\sin^{-1}\left(\sin\frac{\pi}{4}\right) = \frac{\pi}{4}$ by the cancellation equations. OR $\sin^{-1}\left(\sin\frac{\pi}{4}\right) = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$ by work.

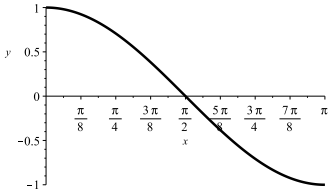
Ex $\sin^{-1}\left(\sin\frac{5\pi}{4}\right) \neq \frac{5\pi}{4}$ BUT $\sin^{-1}\left(\sin\frac{5\pi}{4}\right) = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$

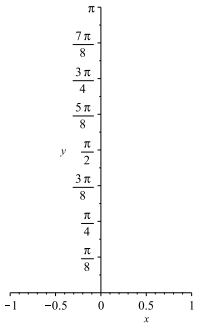
since $\frac{5\pi}{4}$ is not in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ since $-\frac{\pi}{4}$ is in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Ex $\sin\left(\sin^{-1}\frac{1}{2}\right) = \frac{1}{2}$ by the cancellation equations OR $\sin\left(\sin^{-1}\frac{1}{2}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ by work.

Similarly for $f(x) = \cos x$, restrict the domain to $[0, \pi]$ and note that the range is still $[-1, 1]$.







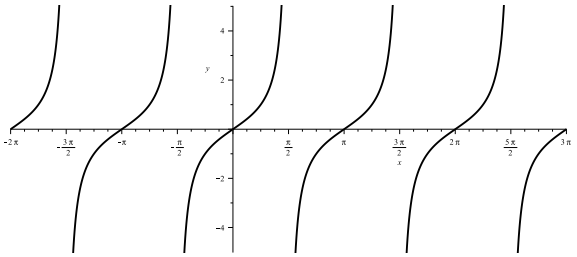
Domain:
Range:

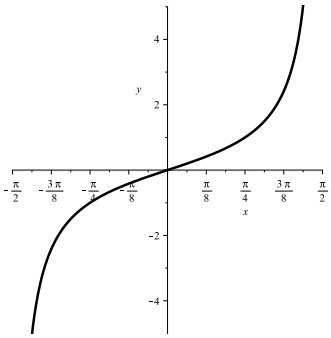
Cancellation Equations

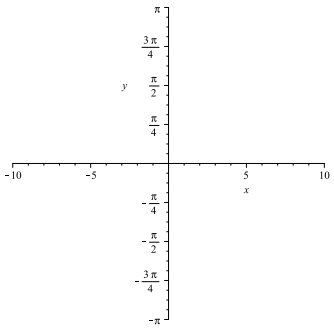
$\cos^{-1}(\cos x) = x$ if $0 \leq x \leq \pi$

$\cos(\cos^{-1} x) = x$ if $-1 \leq x \leq 1$

Similarly for $f(x) = \tan x$, restrict the domain to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and note that the range is $(-\infty, \infty)$.







Domain:
Range:

Cancellation Equations

$\tan^{-1}(\tan x) = x$ if $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

$\tan(\tan^{-1} x) = x$ if $-\infty \leq x \leq \infty$

[See book for graphs of $\sec^{-1}(x), \csc^{-1}(x)$, and $\cot^{-1}(x)$]

Ex $\tan^{-1}(\cos\pi)$

Ex $\sin\left(\cos^{-1}-\frac{\sqrt{3}}{2}\right)$

Ex $\tan^{-1}(\tan\pi)$

Ex Find the exact value of $\cos\left(\arcsin\frac{3}{4}\right)$

Ex Write the following expression as an algebraic expression: $\sin(\tan^{-1}x)$