Ex Suppose $T(x) = f(x)$ represents the temperature in a rod a position x. Suppose we take a temperature measurement and want to know the position based on that temperature.

ie. x is now a function of   .   Mathematically:

Def A function g is the INVERSE FUNCTION of the function f if

Notation: The inverse function is denoted by   Important:

The definition is used to show that 2 functions are inverses of each other – you must show:

(a). Both cancelation equations are satisfied:

(b). The domain and range must interchange:
ie.

Ex Verify the $f(x) = \dfrac{1}{\sqrt{x-2}}$ has the inverse $f^{-1} = \dfrac{1}{x^2} + 2$.   (And find the domain and range of both.)

Since the domain and range interchange $\Longrightarrow$ If the point (a, b) is on f, then the point   is on f^{-1}.

Ex Use this fact to sketch the inverse of $f(x)$

Will a function f always have an inverse? If not, how can we tell?

Ex Sketch $f(x) = x^2 + 1$ and then sketch its reflection through the line $y = x$.

Def A function is called ONE-TO-ONE if

Ex Determine whether the following functions will have an inverse.

Steps for finding and inverse of $f(x)$

0. Verify that an inverse exists.

1. Write $y = f(x)$.

2. Solve for x in terms of y (if possible).

3. Interchange x and y and write $y = f^{-1}(x)$.

4. Define $\text{dom}(f^{-1})$ as the range of f.

Ex Find the inverse function of $f(x) = \sqrt{2x - 3}$