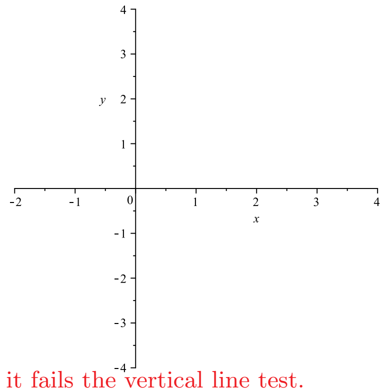

<u>Ex</u> Given $x = \dfrac{3}{t^2+1}$ and $y = t - 1$

Graph the curve on your calculator for $-5 \le t \le 5$.
Use a viewing window $-2 \le x \le 4$ and $-4 \le y \le 4$.

Sketch a copy on the axes to the right.

Then answer the questions below.

(a). Does this curve represent a function? Why or why not? No, it fails the vertical line test.

(b). Could you draw tangent lines to this curve? Yes If so, sketch the tangent line at the point $\left(\frac{3}{2}, 0\right)$.

Give an estimate of the slope of the tangent line that you just drew. Between -0.5 and -1.0 okay

(c). Based on part (b), does it make sense to talk about the derivative $\frac{dy}{dx}$ (or slope) of this curve?
Note: This is the derivative of y with respect to x.

Yes, it makes sense because we can draw tangent lines whose slopes represent the derivative of the curve. Also, the curve itself is drawn in the xy-plane, so it makes sense to talk about the rate of change (or derivative) of y with respect to x.

So given $x = x(t)$ and $y = y(t)$ how do we find $\dfrac{dy}{dx}$?

<u>Solution</u>: Suppose we *could* eliminate t and write $\underline{\quad y = F(x) \quad}$ $(*)$, then $\dfrac{dy}{dx} = \underline{\quad F'(x) \quad}$ $(**)$

But $y = y(t)$ and $x = x(t)$, so $(*)$ becomes $\quad y(t) = F(x(t))$

Differentiate w/ respect to t: $\quad \frac{d}{dt}\left[y(t)\right] = \frac{d}{dt}\left[F(x(t))\right]$

$$y'(t) = F'(x(t)) \cdot x'(t) \qquad \text{using the Chain Rule}$$

$$y'(t) = F'(x) \cdot x'(t) \qquad \text{simplifying notation}$$

$$\Rightarrow \quad F'(x) = \frac{y'(t)}{x'(t)} \qquad \text{solving for } F'(x) \quad (x'(t) \neq 0)$$

Recall, from $(**)$, $\dfrac{dy}{dx} = F'(x) \quad \Rightarrow \quad \dfrac{dy}{dx} = \dfrac{y'(t)}{x'(t)}$

In Leibniz Notation:

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \quad \text{if } dx/dt \neq 0$$

[Note: Formula is still valid even if we can't eliminate t and write $y = F(x)$.]

Ex Given $x = \dfrac{3}{t^2+1}$ and $y = t - 1$

(a). Find $\dfrac{dy}{dx}$.

(b). Find the slope of the tangent line to the curve at the point $\left(\frac{3}{2}, 0\right)$.

(c). Find the equation of the tangent line to the curve at the point $\left(\frac{3}{2}, 0\right)$.

(d). Graph this tangent line on your calculator along with the original curve to verify that it is, in fact, the correct tangent line. [Hint: Remember you need to write the tangent line as a parametric curve.]

(e). Find the equation of the tangent line to the curve at the point corresponding to $t = 0$.

Can we find 2^{nd} derivatives? i.e. How do we find $\dfrac{d^2y}{dx^2}$?

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right]$$

$$= \frac{d}{dx}\left[y'\right]$$

$$= \frac{d(y')}{dx}$$

but y' is parametrically defined (e.g. function of t), so use the formula from p.1 replacing y with y'.

$$= \frac{d(y')/dt}{dx/dt}$$

$$= \frac{\frac{d}{dt}[y']}{dx/dt}$$

$$= \frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{dx/dt}$$

So $\dfrac{d^2y}{dx^2} = \dfrac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{\frac{dx}{dt}}$

Warning!! $\dfrac{d^2y}{dx^2} \neq \dfrac{\frac{d^2y}{dt^2}}{\frac{d^2x}{dt^2}}$

<u>Ex</u> $x = \cos 5t, \quad y = \sin 5t$

(a). Find $\dfrac{dy}{dx}$.

(b). Find $\dfrac{d^2y}{dx^2}$.

Ex $x = t^3 - 6t^2, \qquad y = t^3 - 12t$

(a). Find the points on the curve where the tangent is horizontal or vertical.

(b). For what values of t is the ***curve*** increasing or decreasing?

(c). Use the information obtained in parts (a) and (b) to help sketch the curve on the axes below.