NEW MATERIAL: SECTION 10.1-10.3

1. Given the parametric equations: $x = \ln t, \quad y = 1 + t^2$,

(a). Find dy/dx.

(b). Find d^2y/dx^2.

(c). Find the equation of the tangent line at the point (0,2).

(d). Eliminate the parameter t to find the Cartesian equation of the curve.
Express your answer in the form $y = f(x)$.

2. Given the parametric curve: $x = \sin 2t \;\; y = 4\sin t$ on $0 \le t \le 2\pi$, find all the points where there is a horizontal or vertical tangent line. [You must show all work!]

3. Find the area of the region bounded by the parametric curve $x = 2\cot\theta, y = 2\sin^2\theta$ for $0 < \theta < \pi$.
[Be careful, the curve is traced out from right to left.]

4. Sec. 10.1 #28

5. For each of the polar coordinates $\left(-1, \frac{\pi}{3}\right)$ and $(2, 3\pi)$,

(a). Plot them in the polar coordinate system.

(b). Find the Cartesian coordinate.

6. Sec. 10.3 #54

7. Find a polar equation for the curve given by the Cartesian equation $x + y = 2$

8. Identify the curve by finding a Cartesian equation for the curve $r = 4\sec\theta$

9. Find the slope of the tangent line to the polar curve $r = \sin 3\theta$ at $\theta = \frac{\pi}{6}$.

10. Find the points on the curve $r = 2\cos\theta$ where the tangent line is horizontal or vertical for $0 \le \theta < \pi$.

11. Find the **points** of intersection of the following curves. $r = \sin\theta, \quad r = \sin 2\theta$. (Why is it sufficient to only consider the interval $[0, 2\pi]$?)

<u>THE REMAINDER OF THIS REVIEW COVERS MATERIAL PRIOR TO EXAM 3.</u>

12. Use the geometric series to expand $f(x) = \dfrac{1}{1+2x}$ as a power series.

13. For the function $f(x) = 1 + x + x^2$,

(a). Find the Taylor Series for $f(x)$ centered at $a = 2$.

(b). Expand and simplify your answer to **(a)**. Explain why this simplified expression makes sense.

14. Find the radius of convergence and the exact <u>interval</u> of convergence for the following series.

(a). $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)!\,(x-3)^n}{2^n}$

(b). $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{(x+2)^n}{n2^n}$

15. Explain the Integral Test in your own words. Include sketches to illustrate how the series and the integral are related.

16. Find the <u>**SUM**</u> of the following series or show that it diverges.

(a). $\displaystyle\sum_{n=0}^{\infty} \frac{2^{2n+1}}{5^n}$

(b). $\displaystyle\sum_{n=2}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n}\right)$

17. Determine whether the following series diverge or converge.

(a). $\displaystyle\sum_{n=1}^{\infty} \frac{\sin n}{1+n^2}$

(b). $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{n^3+1}$

(c). $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^2+1}{n^3+1}$

See old Review Sheets and Exam 3 for more practice.

18. Error bounds for alternating series and for series that converge by the Integral Test. [See Exam 3 Review Sheet.]

19. Determine whether the following <u>sequences</u> converge or diverge. Find the limit if it converges.

(a). $a_n = \dfrac{10^n}{9^{n+1}}$

(b). $a_n = \dfrac{2n^3 - 1}{3 + n^3}$

20. Find the derivative.

(a). $f(\theta) = e^{\sin 2\theta} + 3^\theta$

(b). $y = \sin^{-1}(x^2)$

(c). $y = \sinh x$

(d). $y = (\sin x)^{3x}$

21. Find the equation of the tangent line to $y = \log_3 x$ at $x = 9$.

22. Evaluate the following limits:

(a). $\lim\limits_{x\to\infty} (1+2x)^{1/3x}$

(b). $\lim\limits_{x\to 0} \dfrac{\sin x}{x^2}$

23. Evaluate the following integrals:

(a). $\displaystyle\int \sec^4 x \tan^2 x \, dx$

(b). $\displaystyle\int \frac{1}{x(\ln x)^2} \, dx$

(c). $\displaystyle\int \frac{4x}{(x^2-1)(x^2+1)} \, dx$

(d). $\displaystyle\int \frac{x}{x^2-x} \, dx$

(e). $\displaystyle\int_0^2 \frac{1}{x^2-1} \, dx$

(f). $\displaystyle\int_0^{2\sqrt{3}} \frac{x^3}{\sqrt{16-x^2}} \, dx$

(g). $\displaystyle\int \frac{1}{x^3-4x^2+4x} \, dx$

(h). $\displaystyle\int 4^t \, dt$

24. Find the area under the curve $\dfrac{1}{x^2+16}$ for $0 \le x \le 3$.

25. Integrate $\displaystyle\int_0^{\infty} xe^{ax} \, dx$ for $a \neq 0$ and determine for which values of a it converges.

26. Section 6.5 #3

27. Use Simpson's Rule with $n = 4$ to approximate the value of $\displaystyle\int_4^6 \frac{1}{x^2} \, dx$.
[Do **NOT** simplify.]

28. Use the Trapezoid Rule with $n = 6$ to approximate the value of $\displaystyle\int_0^{10} \sin(x^2) \, dx$. [Do **NOT** simplify.]

29. Let g denote the inverse function of f i.e. $g = f^{-1}$. Given $f(x) = 3x + \cos 2x$ on $0 \le x \le \dfrac{\pi}{2}$, find $g'(1)$.

30. Find the exact value of the following:

(a). $\sin\left(\arctan \dfrac{5}{4}\right)$

(b). $\arctan\left(\sin \dfrac{3\pi}{2}\right)$

(c). $\sin^{-1}\left(\sin \dfrac{5\pi}{4}\right)$

31. Solve the following equations for x. [Simplify your answers.]

(a). $\ln 2 + \ln(x-3)$

(b). $e^{x^2+x} = 1$

[Look at previous exams, quizzes, and review sheets for more review.]