Name: ______________________________ **Exam 3**

Math 152, Calculus II – Crawford **05 May 2015**

Score	
1	/8
2	/16
3	/12
4	/12
5	/24
6	/10
7	/12
8	/10
Total	/100

- Calculators, books, or notes (in any form) are <u>not</u> allowed.
- You may use the formulas found on the last page.
- Clearly indicate your answers.
- ***Show all your work*** – partial credit may be given for written work.
- Good Luck!

1. (8 pts). Determine whether the following <u>**series**</u> converges or diverges. <u>**If it is convergent, find its sum**</u>. **[Show all your work.]**

$$\sum_{n=1}^{\infty} \frac{2 \cdot 3^{1-n}}{2^{n+2}}$$

2. (16 pts). Determine whether the following series converge or diverge. [Show all your work and clearly indicate any tests that you use.]

(a). $\displaystyle\sum_{n=1}^{\infty} \frac{n^{2n}}{(2n^2+1)^n}$

(b). $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{3/2}}$

3. (12 pts). Determine whether the following series converges absolutely, converges conditionally, or diverges. [Show all your work and clearly indicate any tests that you use.]

$$\sum_{n=0}^{\infty}(-1)^n \frac{3n+2}{2n^2-5}$$

4. (12 pts). Given $\sum_{n=0}^{\infty}(-1)^n \frac{1}{n^2+36}$

(a). If s_9 is used to approximate the infinite series, what is the bound for the error R_9 ?
[Leave answer as a fraction.]

(b). How many terms are needed to approximate the infinite series with an error less than 0.01?

(c). TRUE or FALSE If s_9 is used to approximate the series $\sum_{n=0}^{\infty} \frac{1}{n^2+36}$, then the bound on the error $|R_9|$ will be the same as your answer in part (a).

[Don't spend time trying to compute it. You should be able to quickly determine T/F without computing the error.]

5. (24 pts). Find the radius of convergence and interval of convergence for the following series.

(a). $\displaystyle\sum_{n=0}^{\infty} \frac{(x+2)^n}{3^n(n^2+2)}$

(b). $\displaystyle\sum_{n=0}^{\infty} \frac{2n!}{4n+1}x^n$

6. (10 pts). Use the <u>definition</u> of Taylor Series to derive the expansion of $f(x) = e^{3x}$ centered at $\underline{\mathbf{a = 1}}$.

[Write your final answer in concise summation notation and simplify. Do **<u>not</u>** find the radius or interval of convergence.]

7. (12 pts). [For this problem, you may write your final answers expanded **<u>or</u>** in concise summation notation.]

(a). Use a known Maclaurin Series to obtain the Maclaurin Series for $f(x) = \dfrac{\sin(x^2)}{x}$.
[You <u>must</u> simplify your answer before answering part (b).]

(b). Use the (simplified) series obtained in part (a) to evaluate the following integral $\displaystyle\int \frac{\sin(x^2)}{x}\, dx$

8. (10 pts). True or False and Matching. Determine whether the following statements are true or false.

(a). T F If $\sum_{n=1}^{\infty} c_n x^n$ converges for $x = -4$, but diverges for $x = 8$, then it must converge for $x = 2$.

(b). T F If $\sum_{n=1}^{\infty} c_n x^n$ converges for $x = -4$, but diverges for $x = 8$, then it must diverge for $x = -8$.

(c). T F If $\sum_{n=1}^{\infty} c_n x^n$ converges for $x = -4$, but diverges for $x = 8$, then it must diverge for $x = -10$.

(d). T F If $\lim a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

(e). T F If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} |a_n|$ must also converge.

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = \left(\frac{n+1}{n}\right)^n = e \qquad \lim_{n\to 0}(1+n)^{1/n} = e$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad R = 1 \qquad e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad R = \infty$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \quad R = \infty \qquad \cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \quad R = \infty$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)} \quad R = 1 \qquad \ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} \quad R = 1$$