

1. Given $f(x) = \sin x + 2 \cos x$, $-\frac{\pi}{4} \leq x \leq 0$, find $(f^{-1})'(a)$ for $a = 2$. Ans: $(f^{-1})'(2) = 1$ Note: $f^{-1}(2) = 0$
2. Solve for x : (a). $e^{3x-7} = 6$ $x = \frac{7+\ln 6}{3}$ (b). $\ln(x+1) - \ln x = 1$ $x = \frac{1}{e-1}$
3. Given $f(x) = \ln(\cos x + 2)$
- (a). What is the domain of f ? Domain: all real
- (b). Find the relative extreme values on the interval $[-1,1]$. [Indicate max or min.] Max value of $\ln 3$ at $x = 0$
4. Evaluate the following integrals.
- (a). $\int_0^1 \frac{1}{4-2x} dx = -\frac{1}{2} \ln|4-2x| \Big|_0^1 = \frac{1}{2} \ln 2$ (f). $\int \operatorname{sech}^2(5x) dx = \frac{1}{5} \tanh(5x) + C$
- (b). $\int e^{-7x} - \frac{7 \ln x}{x} dx = -\frac{1}{7} e^{-7x} - \frac{7}{2} (\ln x)^2 + C$ (g). $\int \sec(5x) dx = \frac{1}{5} \ln |\sec(5x) + \tan(5x)| + C$
- (c). $\int \frac{3x^2 + 2x}{x^3 + x^2 + 1} dx = \ln|x^3 + x^2 + 1| + C$ (h). $\int_0^{\ln 2} \frac{e^{3x} + 1}{e^x} dx = \frac{1}{2} e^{2x} - e^{-x} \Big|_0^2 = 2$
Simplify using log./exp. properties.
- (d). $\int \frac{3x}{\sqrt{1-36x^2}} dx = -\frac{1}{12} \sqrt{1-36x^2} + C$
- (e). $\int \frac{3}{\sqrt{1-36x^2}} dx = \frac{1}{2} \sin^{-1}(6x) + C$ (i). $\int \frac{\tan x}{\ln(\cos x)} dx = -\ln |\ln(\cos x)| + C$
5. Given the function $f(x) = e^{-x^2}$
- (a). Evaluate $\lim_{x \rightarrow -\infty} e^{-x^2} = 0$ (b). Evaluate $\lim_{x \rightarrow +\infty} e^{-x^2} = 0$ (c). Find $f'(x)$ $f'(x) = -2xe^{-x^2}$
6. What is the formula for $\log_a x$ in terms of the natural logarithmic function? $\log_a x = \frac{\ln x}{\ln a}$
7. Find the exact value of (a). $\sin(2 \tan^{-1}(1)) = 1$ (b). $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right) = \frac{5\pi}{6}$
8. Simplify the following expression so that it is an algebraic expression of x : $\cos(\sin^{-1}(2x)) = \sqrt{1-4x^2}$

9. Differentiate the following functions:

(a). $y = x^4 - 4^x + e^{4x} + \ln 4x$
 $y' = 4x^3 + 4^x \cdot \ln 4 + 4e^{4x} + \frac{1}{x}$

(d). $h(\theta) = 3 \ln \left(\frac{2 + \cos \theta}{\theta^2} \right)$ $h'(\theta) = \frac{-3 \sin \theta}{2 + \cos \theta} - \frac{6}{\theta}$

(b). $y = x^{\frac{1}{x}}$ $y' = \frac{1}{x^2}(1 - \ln x) \cdot x^{1/x}$

(e). $y = \cosh(\ln(x^3 + 2x^2))$
 $y' = \frac{3x^2 + 4x}{x^3 + 2x^2} \sinh(\ln(x^3 + 2x^2))$

(c). $y = \pi^x - \ln e^x$ $y' = \pi^x \cdot \ln \pi - 1$

(f). $f(x) = x \sin^{-1}(3x)$ $f'(x) = \frac{3x}{\sqrt{1-9x^2}} + \sin^{-1}(3x)$

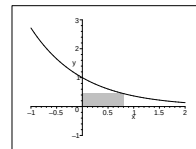
10. Find the equation of the tangent line to $y = \log_3 x$ at $x = 1$

$$y = \frac{1}{\ln 3}(x - 1)$$

11. (a). Find the maximum area of a rectangle in the first quadrant with 2 sides on the x - and y -axes and one vertex on the curve $y = e^{-x}$. See the figure below. [Hint: Express the area of such a rectangle in terms of x only.]

Maximize $A = xy$ subject to $y = e^{-x}$

$\Rightarrow x = 1$. So max area = $\frac{1}{e}$



(b). Sketch the picture for a rectangle in the first quadrant with 2 sides on the x - and y -axes and one vertex on the curve $y = e^x$. Without using Calculus, determine whether there exists such a rectangle with a maximum area. Briefly explain (a couple of sentences) why or why not. No, if you draw the picture you will see that you can always create a bigger rectangle by taking x further out.

12. Given that a population follows the law of exponential growth, $y(t) = Ce^{kt}$ where y is the population and t is time in years.

(a). Find the proportional constant k , if the population triples every 50 years. $k = \frac{\ln 3}{50}$

(b). If the population is 100 after 5 years, find the population at time t . $y = \frac{100}{\sqrt[10]{3}} e^{\frac{\ln 3}{50} t} = 89.60e^{.02197t}$

13. Find the following limits. You must clearly show all steps and indicate where you use L'Hopital's Rule.

(a). $\lim_{x \rightarrow 0} \frac{\sin 4x}{2x} = 2$ (b). $\lim_{x \rightarrow \infty} (1 + x)^{\frac{1}{x}} = 1$ (c). $\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 3x + 2} = 1$ (d). $\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 3x - 2} = 0$