

1. Given $f(x) = \sin x + 2 \cos x$, $-\frac{\pi}{4} \leq x \leq 0$, find $(f^{-1})'(a)$ for $a = 2$.

2. Solve for x : (a). $e^{3x-7} = 6$ (b). $\ln(x+1) - \ln x = 1$

3. Given $f(x) = \ln(\cos x + 2)$

(a). What is the domain of f ?

(b). Find the relative extreme values on the interval $[-1, 1]$. [Indicate max or min.]

4. Evaluate the following integrals.

(a). $\int_0^1 \frac{1}{4-2x} dx$

(f). $\int \operatorname{sech}^2(5x) dx$

(b). $\int e^{-7x} - \frac{7 \ln x}{x} dx$

(g). $\int \sec(5x) dx$

(c). $\int \frac{3x^2 + 2x}{x^3 + x^2 + 1} dx$

(h). $\int_0^{\ln 2} \frac{e^{3x} + 1}{e^x} dx$
Simplify using log./exp. properties.

(d). $\int \frac{3x}{\sqrt{1-36x^2}} dx$

(e). $\int \frac{3}{\sqrt{1-36x^2}} dx$

(i). $\int \frac{\tan x}{\ln(\cos x)} dx$

5. Given the function $f(x) = e^{-x^2}$

(a). Evaluate $\lim_{x \rightarrow -\infty} e^{-x^2}$ (b). Evaluate $\lim_{x \rightarrow +\infty} e^{-x^2}$ (c). Find $f'(x)$

6. What is the formula for $\log_a x$ in terms of the natural logarithmic function?

7. Find the exact value of (a). $\sin(2 \tan^{-1}(1))$ (b). $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$

8. Simplify the following expression so that it is an algebraic expression of x : $\cos(\sin^{-1}(2x))$

9. Differentiate the following functions:

(a). $y = x^4 - 4^x + e^{4x} + \ln 4x$

(d). $h(\theta) = 3 \ln \left(\frac{2 + \cos \theta}{\theta^2} \right)$

(b). $y = x^{\frac{1}{x}}$

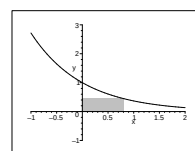
(e). $y = \cosh(\ln(x^3 + 2x^2))$

(c). $y = \pi^x - \ln e^x$

(f). $f(x) = x \sin^{-1}(3x)$

10. Find the equation of the tangent line to $y = \log_3 x$ at $x = 1$.

11. (a). Find the maximum area of a rectangle in the first quadrant with 2 sides on the x - and y -axes and one vertex on the curve $y = e^{-x}$. See the figure below. [Hint: Express the area of such a rectangle in terms of x only.]



(b). Sketch the picture for a rectangle in the first quadrant with 2 sides on the x - and y -axes and one vertex on the curve $y = e^x$. Without using Calculus, determine whether there exists such a rectangle with a maximum area. Briefly explain (a couple of sentences) why or why not.

12. Given that a population follows the law of exponential growth, $y(t) = Ce^{kt}$ where y is the population and t is time in years.

(a). Find the proportional constant k , if the population triples every 50 years.

(b). If the population is 100 after 5 years, find the population at time t .

13. Find the following limits. You must clearly show all steps and indicate where you use L'Hopital's Rule.

(a). $\lim_{x \rightarrow 0} \frac{\sin 4x}{2x}$

(b). $\lim_{x \rightarrow \infty} (1 + x)^{\frac{1}{x}}$

(c). $\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 3x + 2}$

(d). $\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 3x - 2}$