

INFORMATION FROM THE FUNCTION $y = f(x)$:**1. Domain:**

Find the domain of the function by determining all values of x for which the function $f(x)$ is defined.

2. Intercepts:

(a). x -intercept(s): Set $y = 0$ and solve for x . [May have more than one]

(b). y -intercept: Set $x = 0$ and find the y -value, i.e. $f(0)$. [Only one]

3. Asymptotes:

(a). Vertical asymptotes: If the $\lim_{x \rightarrow a} f(x) = \frac{\text{nonzero number}}{0} = +\infty$ or $-\infty$, then a line of the form $x = a$ is a vertical asymptote. [Note: The function is undefined at $x = a$.]

(b). Horizontal asymptotes: If the $\lim_{x \rightarrow \infty} f(x) = L$ or $\lim_{x \rightarrow -\infty} f(x) = L$, then a line of the form $y = L$ is a horizontal asymptote.

(c). Slant asymptotes: Slant asymptotes are lines of the form $y = mx + b$ that occur in rational functions $f(x) = \frac{p(x)}{q(x)}$ where the degree of the numerator is one higher than the degree of the denominator. Note in this case, the function will be growing or decreasing without bound (i.e. $\lim_{x \rightarrow \pm\infty} f(x) = +\infty$ or $-\infty$).

(i) Use long division to rewrite the function as $f(x) = mx + b + \frac{\text{remainder}}{\text{denominator}}$.

(ii) Verify that $\lim_{x \rightarrow \pm\infty} \frac{\text{remainder}}{\text{denominator}} = 0$.

(iii) Hence, as $x \rightarrow \pm\infty$, the function looks like the line $y = mx + b$ which is the slant asymptote.

INFORMATION FROM THE FIRST DERIVATIVE $f'(x)$:**4. Intervals of Increase or Decrease:**

To find intervals where the function is increasing or decreasing,

- (i). Find the critical numbers (i.e. x -values where $f'(x) = 0$ or $f'(x)$ is undefined).
- (ii). Draw a number line and separate it into intervals using the critical numbers and asymptotes.
- (iii). Choose a number from each interval and find the value of f' at that number.
- (iv). If f' is negative, then f is decreasing on that interval.
If f' is positive, then f is increasing on that interval.

5. Local Maximum and Minimum Values:

- (a). A local maximum occurs at any number x in the domain where f is increasing on the left of that number and decreasing on the right of that number.
- (b). A local minimum occurs at any number x in the domain where f is decreasing on the left of that number and increasing on the right of that number.
- (c). Plug these x -numbers into the original $f(x)$, to find the local minimum and maximum values of f .

INFORMATION FROM THE SECOND DERIVATIVE $f''(x)$:

6. Intervals of Concave Up or Concave Down:

To find intervals where the function is concave up or concave down,

- (i). Find the interesting numbers (i.e. x -values where $f''(x) = 0$ or $f''(x)$ is undefined).
- (ii). Draw a number line and separate it into intervals using these interesting numbers and asymptotes.
- (iii). Choose a number from each interval and find the value of f'' at that number.
- (iv). If f'' is negative, then f is concave down on that interval.
If f'' is positive, then f is concave up on that interval.

7. Inflection Points:

- (a). A *inflection point* occurs at any number x in the domain where f is concave up on the left of that number and concave down on the right side of that number or vice versa.
- (b). Plug these x -numbers into the original $f(x)$, to find the actual inflection point of f .

FINALLY:

8. Sketch the Curve:

Use all the information obtained above to sketch the curve:

- (i). Draw asymptotes from part (3) with dashed lines.
- (ii). Plot the intercepts, maximum and minimum values, and inflection points found in (2), (5), & (7).
- (iii). Sketch a curve that passes through these points that increases and decreases as found in part (4), with concavity as found in part (6), and approaches the asymptotes from part (3).