1. For each pair of functions f and g, find the composite function  $F = f \circ g$ .

[You do not need to simplify.]

(a).  $f(x) = x^4 + x^2$ ,  $g(x) = 3x - 2x^5$ 

**(b).**  $f(x) = x^8, g(x) = \frac{\sin x}{x-1}$ 

(c).  $f(x) = \sqrt{x}, g(x) = \tan x$ 

(d).  $f(x) = \tan x, g(x) = \sqrt{x}$ 

- **2.** For each function F below, find a pair of functions f and g such that  $F = f \circ g$ .
- (a).  $F(x) = (6x^2 2x + 3)^2 4$

**(b).**  $F(x) = \left(\frac{5-3x}{x+2}\right)^9$ 

(c).  $F(x) = 3\sin x - \sqrt{\sin x}$ 

(d).  $F(x) = \tan(\pi x + 1)$ 

- **3.** A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius (in inches) of the outer circle is given by r(t) = 10t, where t is the time (in seconds) after the pebble strikes the water. The area of a circle is given by the function  $A(r) = \pi r^2$ .
- (a). Find  $(A \circ r)(t) = A(r(t))$
- (b). Fill in the blank to explain in words what it means  $A \circ r$  means: The expression from part (a)

A = gives the \_\_\_\_\_ of the outer circle as a function of \_\_\_\_\_ .

- **4.** Let  $f(x) = x^2 + 5x 3$  and  $g(x) = 3x^2 + 2x$ .
- (a). Find  $F(x) = (f \circ g)(x)$ . Simplify/expand your answer.

(b). Find F'(x).

(c). Find f'(x) and g'(x).

(d). Find f'(g(x)) [i.e. the composition of  $f' \circ g$ ].

(e). Find  $f'(g(x)) \cdot g'(x)$  and simplify your answer.

(f). Compare the result of part (b) with part (e):

True or False:

$$F'(x) = f'(g(x)) \cdot g'(x)$$

- 5. Suppose the graphs on the next page are given for a car company where
- c(w) = number of cars produced by w workers and
- p(c) = profit in dollars from producing c cars.
- (a). Let  $P(w) = (p \circ c)(w) = p(c(w)) = \text{profit from } w \text{ workers.}$ 
  - (i) If there are w = 200 workers, how many cars c are produced?
  - (ii) If c is the number of cars found in part (i), what is the profit p?
  - (iii) Use parts (i) and (ii) to determine the profit P when you have 200 workers, i.e. find  $P(200) = (p \circ c)(200) = p(c(200))$ .
  - (iv) Repeat parts (i)-(iii) to find P(0), P(100) and P(300).
  - (v) Use the results of (iii)-(iv) to complete the following table.

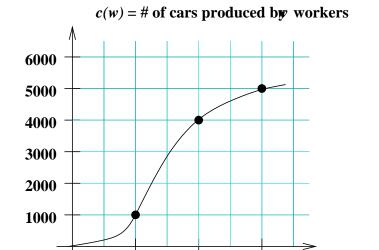
w	P(w)
0	
100	
200	
300	

- (vi) Use the table to sketch the graph for  $P(w) = (p \circ c)(w) = p(c(w))$  on following page.
- (b). If c'(200) = 10, then the slope of the tangent line at w = 200 is \_\_\_\_\_\_ . So if 200 workers are currently working, approximately how many more cars will be produced by adding one more worker?
- (c). If p'(4000) = 450, then the slope of the tangent line at c = 4000 is \_\_\_\_\_\_. So if 4000 cars are currently being produced, approximately how much more profit will be made by producing one more car?
- (d). Based on your answers to parts (b) and (c), fill in the following blanks:

  If 200 workers are currently working and you add more workers, it will result in \_\_\_\_\_ more cars per worker and \_\_\_\_\_ profit per car. So the overall increase to profit is [Fill in the correct numbers below.]

$$\frac{\text{cars}}{\text{worker}} \times \frac{\text{profit(\$)}}{\text{cars}} = \frac{\text{profit(\$)}}{\text{workers}}$$

- (e). Based on your answer to part (d), P'(200) =\_\_\_\_\_. [i.e. The change in profit P(w) from adding one more worker to the current 200 working.]
- (f). Use your answers from (d) and (e) to write a relationship between P'(200), c'(200), and p'(4000).



200

**300** 

w = # of worker

100

