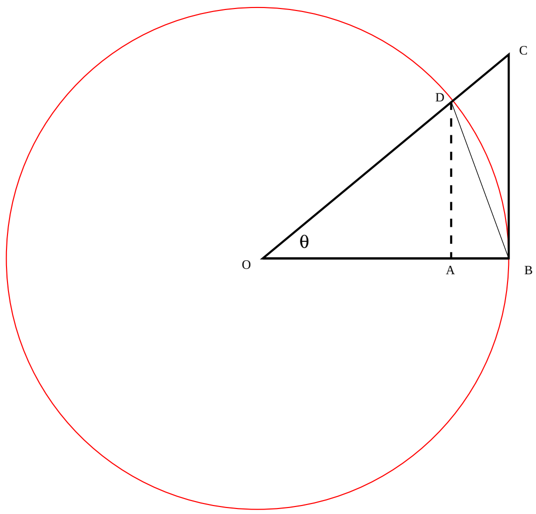
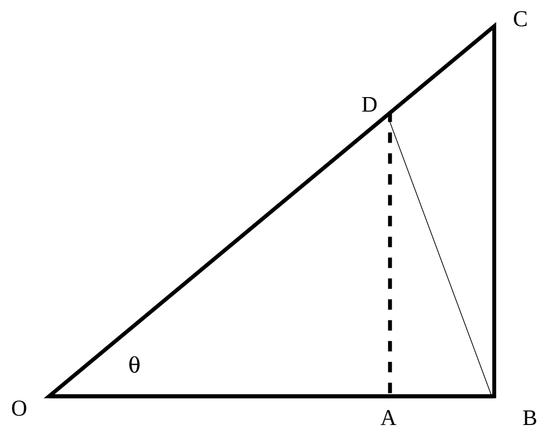
Consider the unit circle and an angle θ in quadrant I as shown in the figure on the left. The figure on the right is just a larger picture of the triangles formed with this angle and points on the circle.





1. Answer the following questions based on the above graphs. Use your answers to label the graph.

(a). What is the length of the radius? What is the length of side OB? What is the length of side OD?

$r =$ $\overline{OB} =$ $\overline{OD} =$

Label these lengths on the pictures above.

(b). Label the length of segment OA as x and the height of segment AD as y.

In other words the point D has coordinates (x, y).

But since the point D lies on the unit circle, we know that

the x-coordinate is $\cos\theta$ and the y-coordinate is __________ Fill in the blanks.

Hence, the length $\overline{OA} = x =$ __________ and the height $\overline{AD} = y =$ __________.

Label these lengths on the pictures above.

(c). Note that triangle OAD is similar to triangle OBC.

Hence, the ratio of their side lengths will be equal:

$$\frac{\text{length of } BC}{\text{length of } OB} = \frac{\text{length of } AD}{\text{length of } OA} \qquad \text{i.e.} \quad \frac{\overline{BC}}{\overline{OB}} = \frac{\overline{AD}}{\overline{OA}}$$

Since you know the lengths of OB, AC, and OA from parts (a) and (b), you can use this ratio to find the length of BC. Your answer will contain trigonometric functions. Simplify your answer.

$\overline{BC} = \dfrac{\overline{AD}}{\overline{OA}} \cdot \overline{OB} =$ Label this length on the pictures above.

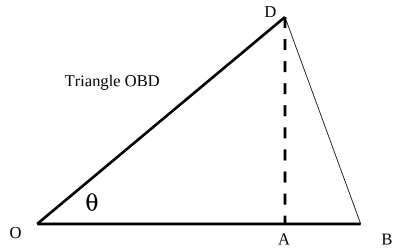
2.

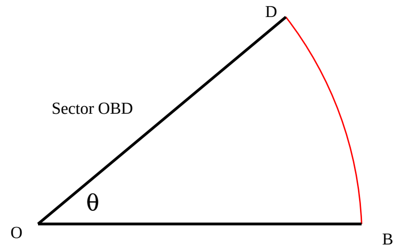
(a). Recall that the area of a triangle is given by the formula $A = \frac{1}{2}(base)(height)$.

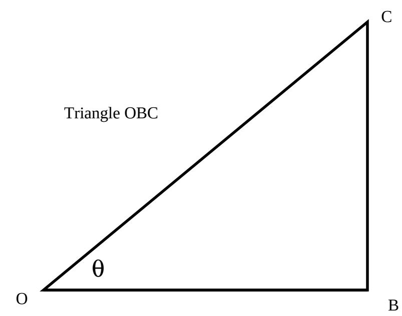
What is the formula for the area of a sector (e.g. "pie slice") of a circle? [Hint: Look in the front of your book.]

$A =$

(b). Label the following pictures with the lengths for OB, AD, and BC found in #1.
(Remember, $\overline{AD}$ and $\overline{BC}$ involve trig. functions.)







A =

A =

A =

(c). Using the lengths you have labeled, find the area of each of the figures above. Simplify your answers (they will involve θ.).

(d). Write the areas found above in order of smallest to largest. If needed, look at the original picture on page 1.]

3. Use the inequality above and The Squeeze Theorem to prove that $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$.

[Note: To use The Squeeze Theorem, we want the expression $\frac{\sin\theta}{\theta}$ to be in between two other expressions for which we know the limit.]

[Continued ⟶]

Start with what we know and then use correct mathematical steps to get what we want:

$$\frac{1}{2}\sin\theta \leq \frac{1}{2}\theta \leq \frac{1}{2}\tan\theta$$

$$\sin\theta \leq \theta \leq \frac{\sin\theta}{\cos\theta}$$

$$1 \geq \frac{\sin\theta}{\theta} \geq \cos\theta$$

$$\lim_{\theta\to 0}\cos\theta \leq \lim_{\theta\to 0}\frac{\sin\theta}{\theta} \leq \lim_{\theta\to 0} 1$$

$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$$

4. Evaluate

$$\begin{aligned}
\lim_{\theta\to 0}\frac{\cos\theta - 1}{\theta} &= \lim_{\theta\to 0}\frac{\cos\theta - 1}{\theta}\cdot \qquad\qquad = \lim_{\theta\to 0}\frac{\cos^2\theta - 1}{\theta(\cos\theta + 1)} \\
&= \lim_{\theta\to 0}\frac{}{\theta(\cos\theta + 1)} = \lim_{\theta\to 0}\frac{\sin\theta}{\theta}\cdot\frac{\sin\theta}{\cos\theta + 1} \\
&= \lim_{\theta\to 0}\frac{\sin\theta}{\theta}\cdot \\
&=
\end{aligned}$$

Two important limits:

5. Use an addition or subtraction formula/identity to rewrite [Hint: Look in the front or back of your book.]

$\sin(x + h) =$