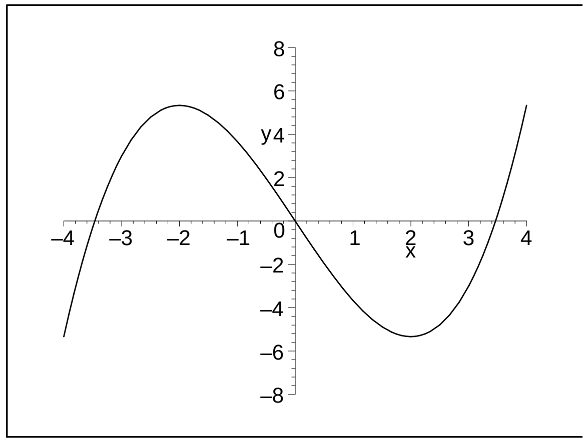
Recall,

$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ gives the derivative of f at the point $x = a$.

For the function

$$f(x) = \frac{1}{3}x^3 - 4x$$

Graph of $f(x) = \frac{1}{3}x^3 - 4x$



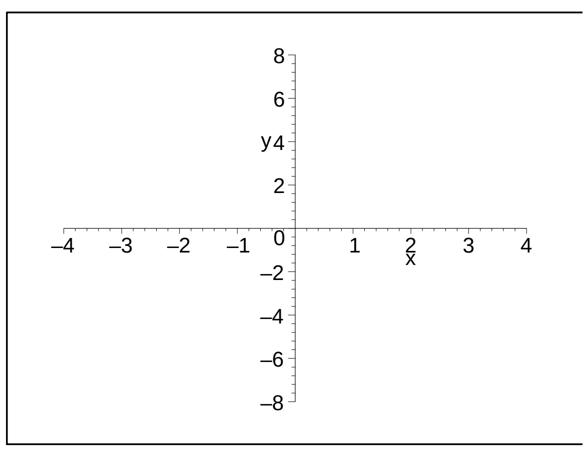
the derivative at $x = a$ is

$$f'(a) = a^2 - 4$$

Details on back.

1. Complete the following table to find the derivative (slope of tangent line) at different values of $x = a$.

$x = a$	$f'(a) = a^2 - 4$
-3	5
-2	
-1	
0	
1	
2	
3	



2. Plot the coordinate pairs $(a, f'(a))$ from the table on the set of axes above.

3. Does it seem like you could connect these points to make a “nice” graph? If so, do it.

$$\boxed{f(x) = \frac{1}{3}x^3 - 4x}$$

Then the derivative at a point $x = a$ found by:

$$\begin{aligned}
f'(a) &= \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} \\
&= \lim_{h\to 0} \frac{\frac{1}{3}(a+h)^3 - 4(a+h) - \left(\frac{1}{3}a^3 - 4a\right)}{h} \\
&= \lim_{h\to 0} \frac{\frac{1}{3}(a^3 + 3a^2h + 3ah^2 + h^3) - 4a - 4h - \frac{1}{3}a^3 + 4a}{h} \\
&= \lim_{h\to 0} \frac{\frac{1}{3}a^3 + a^2h + ah^2 + \frac{1}{3}h^3 - 4a - 4h - \frac{1}{3}a^3 + 4a}{h} \\
&= \lim_{h\to 0} \frac{\frac{1}{3}\cancel{a^3} + a^2h + ah^2 + \frac{1}{3}h^3 - \cancel{4a} - 4h - \frac{1}{3}\cancel{a^3} + \cancel{4a}}{h} \\
&= \lim_{h\to 0} \frac{a^2h + ah^2 + \frac{1}{3}h^3 - 4h}{h} \\
&= \lim_{h\to 0} \frac{h(a^2 + ah + \frac{1}{3}h^2 - 4)}{h} = \lim_{h\to 0} \frac{\cancel{h}(a^2 + ah + \frac{1}{3}h^2 - 4)}{\cancel{h}} \\
&= \lim_{h\to 0} a^2 + ah + \frac{1}{3}h^2 - 4 \\
&= a^2 - 4
\end{aligned}$$

i.e

$$\boxed{f'(a) = a^2 - 4}$$