Name: Key

Math 151 Calculus I – Crawford

Quiz 4
09 May 2017

Books, notes (in any form), and calculators are not allowed. *Show all your work.* Good Luck!

1. (4 pts) Evaluate the following integral.

$$\int \cos^5 x \, \sin x \, dx$$

$u = \cos x$

$du = -\sin x \, dx$

$-du = \sin x \, dx$

$$-\int u^5 \, du$$

$$= -\frac{1}{6} u^6 + C = \boxed{-\frac{1}{6}\cos^6 x + C}$$

2. (5 pts) Evaluate the following integral. ***Simplify your answer.***

$$\int_0^2 \frac{3x}{\sqrt{x^2+4}} \, dx$$

$$= \int_0^2 \frac{3}{\sqrt{x^2+4}} \underbrace{x \, dx}_{\frac{1}{2}du}$$

$u = x^2 + 4$

$du = 2x \, dx$

$\frac{1}{2}du = x \, dx$

$$\frac{1}{2}\int_{x=0}^{x=2} \frac{3}{\sqrt{u}} \, du$$

$$\frac{1}{2}\int_{x=0}^{x=2} 3u^{-1/2} \, du$$

$$\frac{3}{2}\cdot\frac{2}{1} u^{1/2} \Big|_{x=0}^{x=2}$$

$$3(x^2+4)^{1/2} \Big|_0^2$$

$\rightarrow = 3\sqrt{2^2+4} - 3\sqrt{0^2+4}$

$= 3\sqrt{8} - 3\sqrt{4}$

$= 3 \cdot 2\sqrt{2} - 3 \cdot 2$

$= \boxed{6\sqrt{2} - 6}$

Note: If the bounds are converted to u:

$x = 2 \Rightarrow u = 2^2 + 4 = 8$

$x = 0 \Rightarrow u = 0^2 + 4 = 4$

$$\frac{1}{2}\int_4^8 \frac{3}{\sqrt{u}} \, du$$

$$= \frac{3}{2}\int_4^8 u^{-1/2} \, du$$

$$= \frac{3}{2}\cdot\frac{2}{1} u^{1/2} \Big|_4^8$$

$$= 3\sqrt{u} \Big|_4^8$$

$= 3\sqrt{8} - 3\sqrt{4}$

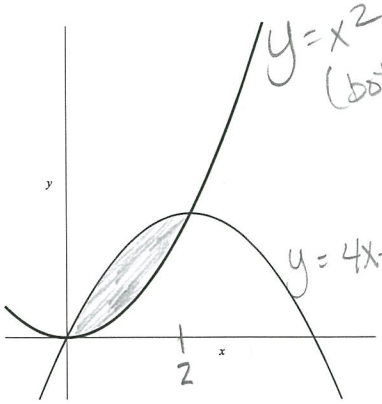
$= 3 \cdot 2\sqrt{2} - 3 \cdot 2$

$= 6\sqrt{2} - 6$

3. (6 pts) Given the graphs of $y = x^2$ and $y = 4x - x^2$ below,

(a). Shade the region enclosed by the two curves.

(b). Find the area of the region enclosed by the two curves.



Intersect: $x^2 = 4x - x^2$

$\Rightarrow 2x^2 - 4x = 0$

$2x(x-2) = 0$

$x = 0, 2$

$$A = \int_0^2 (4x - x^2) - (x^2)\, dx$$

$$= \int_0^2 4x - 2x^2\, dx$$

$$= 2x^2 - \frac{2}{3}x^3 \Big|_0^2$$

$$= 2(2)^2 - \frac{2}{3}(2)^3 - (0)$$

$$= 8 - \frac{16}{3}$$

$$= \frac{24-16}{3} = \boxed{\frac{8}{3}}$$