Books, notes (in any form), calculators, etc., are not allowed. You must *show all your work* for full credit. Good Luck!

1. (15 pts) Given the following function and its derivatives

$$f(x) = \frac{3x^2}{2(x-2)^2} = \frac{3x^2}{2x^2 - 8x + 8} \qquad f'(x) = \frac{-6x}{(x-2)^3} \qquad f''(x) = \frac{12(x+1)}{(x-2)^4}$$

(a). Fill in the following information about the function and its graph. Show all work and write "none", if applicable.

domain: _All real, except $x \neq 2$_

x-intercept(s): _$(0,0)$_

y-intercept: _$(0,0)$_

vertical asymptote(s): _$X = 2$_

horizontal asymptote(s): _$y = \frac{3}{2}$_

slant asymptote: _None_

critical numbers: _$X = 0$_

intervals where increasing: _$(0,2)$_

intervals where decreasing: _$(-\infty, 0) \cup (2, \infty)$_

coordinates of local max(s): _None_

coordinates of local min(s): _$(0,0)$_

intervals where concave up: _$(-1, 2) \cup (2, \infty)$_
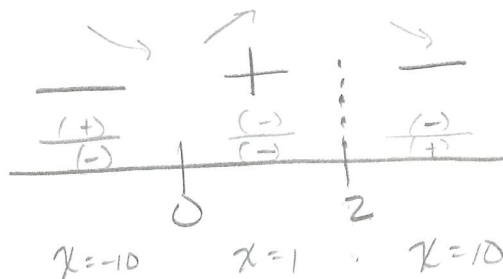
intervals where concave down: _$(-\infty, -1)$_

Inflection Point(s): _$\left(-1, \frac{1}{6}\right)$_

(handwritten work, right side:)

Domain: $2(x-2)^2 \neq 0 \Rightarrow X \neq 2 \leftarrow X = 2$ is V.A.

x-int $(y=0)$: $3x^2 = 0 \Rightarrow X = 0$

y-int $(x=0)$: $f(0) = 0$

H.A.: $\lim\limits_{x \to \pm\infty} \frac{3x^2}{2x^2 - 8x + 8} = \frac{3}{2}$

$f' = 0$
$-6x = 0$
$x = 0$

f' DNE
$(x-2)^3 = 0$
$X = 2 \leftarrow$ V.A. of f.

$f' = \frac{-6x}{(x-2)^3}$

$X = -10 \qquad X = 1 \qquad X = 10$

min at $X = 0 \Rightarrow f(0) = 0$

$f'' = 0$
$12(x+1) = 0$
$X = -1$

f'' DNE
$(x-2)^4 = 0$
$X = 2 \leftarrow$ V.A. of f.

$f'' = \frac{12(x+1)}{(x-2)^4}$
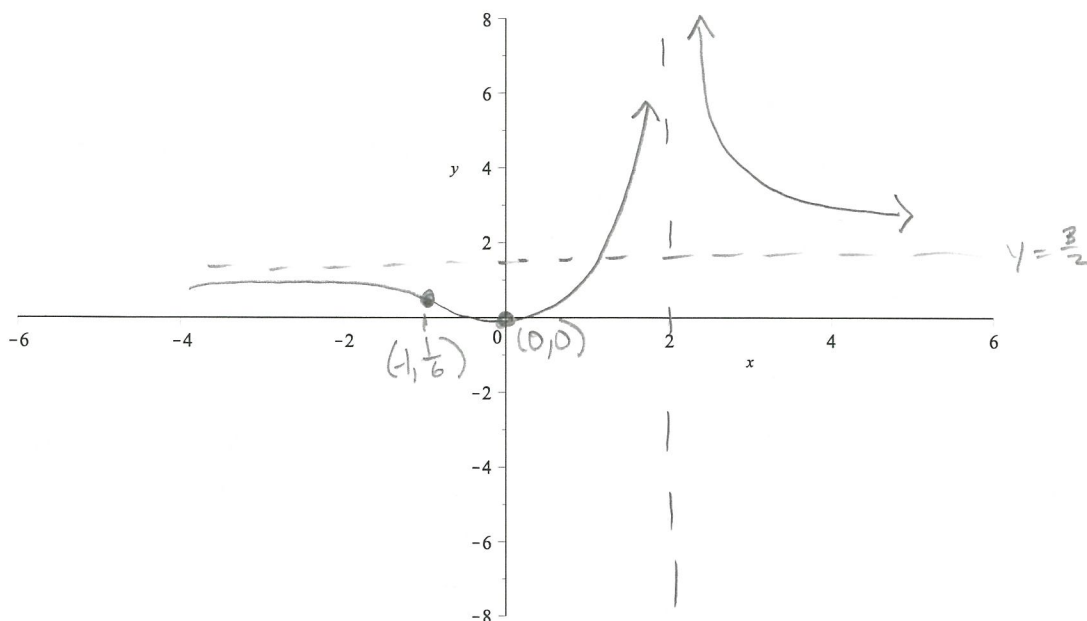

$X = -10 \qquad X = 0 \qquad X = 10$
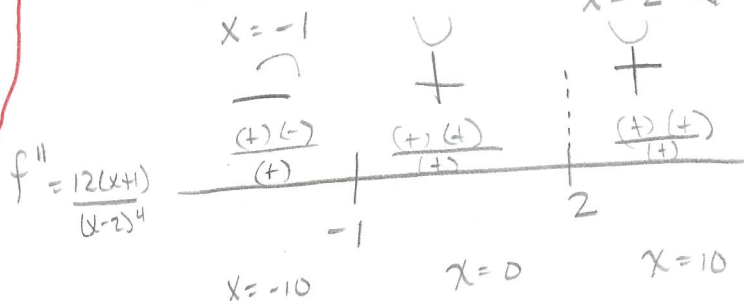
Inf. pt at $X = -1$

$f(-1) = \frac{3(-1)^2}{2(-1)^2 - 8(-1) + 8} = \frac{3}{2 + 8 + 8} = \frac{3}{18} = \frac{1}{6}$

(b). Sketch the graph of the function on the set of axes provided. Label any maximum and minimum values and inflection points.

Just an extra set of axes, in case you need it.