Name: Key

Math 151 Calculus I – Crawford

Take-Home Quiz 2

Due: Thursday, 16 March 2017 by 9:30am

Books, notes, and calculators *are* allowed. You are allowed to work with each other and to get help from the tutors, but you cannot get help from me. ***You must show all your work.*** Good luck! [Scores will be scaled to 15 points after grading.]

1. (6 pts) Solve the following equation. Find only solutions in the interval $[0, 2\pi)$. [Exact solutions.]

$\sin x \tan x + \sin x = 0$

$\sin x(\tan x + 1) = 0$

$\sin x = 0$ or $\tan x = -1$

$$\boxed{x = 0, \pi \qquad x = \frac{3\pi}{4}, \frac{7\pi}{4}}$$

2. (6 pts) Solve the following equation. Find ***(a)*** all solutions and ***(b)*** all solutions in the interval $[0, 2\pi)$. [Exact solutions.]

$4\cos^2(3x) - 3 = 0$

$4\cos^2(3x) = 3$

$\cos^2(3x) = \frac{3}{4}$

$\cos(3x) = \pm\sqrt{\frac{3}{4}}$

$\cos(3x) = \frac{\pm\sqrt{3}}{2}$

$\Rightarrow$

$3x = \frac{\pi}{6} + 2n\pi$

$3x = \frac{5\pi}{6} + 2n\pi$

$3x = \frac{7\pi}{6} + 2n\pi$

$3x = \frac{11\pi}{6} + 2n\pi$

$\Rightarrow$ (a)

$x = \frac{\pi}{18} + \frac{2}{3}n\pi$

$x = \frac{5\pi}{18} + 2n\pi$

$x = \frac{7\pi}{18} + 2n\pi$

$x = \frac{11\pi}{18} + 2n\pi$

$n = 0, \pm1, \pm2, \ldots$

(b)

$n = 0$: $\frac{\pi}{18}, \frac{5\pi}{18}, \frac{7\pi}{18}, \frac{11\pi}{18}$

$n = 1$: $\frac{13\pi}{18}, \frac{17\pi}{18}, \frac{19\pi}{18}, \frac{23\pi}{18}$ (add $\frac{2\pi}{3} = \frac{12\pi}{18}$ to each value)

$n = 2$: $\frac{25\pi}{18}, \frac{29\pi}{18}, \frac{31\pi}{18}, \frac{35\pi}{18}$ (add another $\frac{12\pi}{18}$)

Note $n = 3$: Adds $\frac{2}{3}(3)\pi = 2\pi \Rightarrow$ So all too large

3. (6 pts) Find the derivative of the function $g(x) = x\left(\dfrac{3x^2+1}{2x^3-5x}\right)^5$ [Do not simplify.]

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$$g'(x) = x \cdot 5\left(\frac{3x^2+1}{2x^3-5x}\right)^4 \cdot \frac{d}{dx}\left[\frac{3x^2+1}{2x^3-5x}\right] + \left(\frac{3x^2+1}{2x^3-5x}\right)^5 \cdot 1$$

$$= \boxed{x \cdot 5\left(\frac{3x^2+1}{2x^3-5x}\right)^4 \cdot \frac{(2x^3-5x)(6x)-(3x^2+1)(6x^2-5)}{(2x^3-5x)^2} + \left(\frac{3x^2+1}{2x^3-5x}\right)}$$

4. (6 pts) Find the <u>first *and* second derivatives</u> of $y = \sec(4\theta)$. [Do not simplify the 2nd derivative.]

$$\boxed{y' = \sec(4\theta)\tan(4\theta)\cdot 4 = 4\sec(4\theta)\tan(4\theta)}$$

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$$y'' = 4\sec(4\theta)\cdot\frac{d}{d\theta}[\tan(4\theta)] + \tan(4\theta)\cdot\frac{d}{d\theta}[4\sec(4\theta)]$$

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$$= \boxed{4\sec(4\theta)\cdot\sec^2(4\theta)\cdot 4 + \tan(4\theta)\cdot 4\sec(4\theta)\tan(4\theta)\cdot 4}$$

$$= 16\sec^3(4\theta) + 16\sec(4\theta)\tan^2(4\theta)$$

5. (6 pts) Find an equation of the tangent line to $y = \sqrt{5x+6}$ at $x = 2$.

① pt: $y = \sqrt{5(2)+6} = \sqrt{16} = 4 \Rightarrow (2, 4)$

② slope: $y = (5x+6)^{1/2}$

$$y' = \frac{1}{2}(5x+6)^{-1/2}\cdot 5 = \frac{5}{2\sqrt{5x+6}}\Big|_{x=2}$$

$$= \frac{5}{2\sqrt{5\cdot 2+6}} = \frac{5}{2\sqrt{16}} = \frac{5}{2\cdot 4} = \frac{5}{8} = m$$

$$\boxed{y - 4 = \frac{5}{8}(x-2)}$$