Name: Key

Math 151 Calculus I – Crawford

Quiz 1-A/C

21 February 2017

Books, notes (in any form), and calculators are not allowed. *Show all your work.* Good Luck!

1. (4 pts) Evaluate the following limit.

$$\lim_{x\to 1} \frac{2x^2-3x+1}{x^2+2x-3} = \lim_{x\to 1} \frac{(2x-1)\cancel{(x-1)}}{(x+3)\cancel{(x-1)}} = \lim_{x\to 1} \frac{2x-1}{x+3}$$

$$\left(\frac{2(1)^2-3(1)+1}{(1)^2+2(1)-3} \quad \frac{0}{0}\right)$$

$$= \frac{2(1)-1}{1+3}$$

$$= \boxed{\frac{1}{4}}$$

2. (4 pts) Determine whether the following function has a jump, removable, or infinite discontinuity at $x = -2$. [Show work to justify your answer.]

$$f(x) = \frac{x^2+2x}{(x+2)^2}$$

$$\lim_{x\to -2} \frac{x^2+2x}{(x+2)^2} = \lim_{x\to -2} \frac{x\cancel{(x+2)}}{(x+2)^{\cancel{2}\,1}} = \lim_{x\to -2} \frac{x}{x+2}$$

$$\frac{0}{0}$$

$$\to \frac{-2}{0}$$ Infinite Limit

$\Rightarrow$ $\boxed{\text{Infinite Discontinuity}}$

3. (7 pts) Given $f(x) = \sqrt{x}$,

(a). Use the limit definition $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ **OR** $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ to show that the slope of the tangent line at $x = 9$ is $\frac{1}{6}$.

$$f'(9) = \lim_{h \to 0} \frac{f(9+h) - f(9)}{h} = \lim_{h \to 0} \frac{\sqrt{9+h} - \sqrt{9}}{h} = \lim_{h \to 0} \frac{\sqrt{9+h} - 3}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{9+h} - 3}{h} \cdot \frac{\sqrt{9+h} + 3}{\sqrt{9+h} + 3} = \lim_{h \to 0} \frac{(\sqrt{9+h})^2 - (3)^2}{h(\sqrt{9+h} + 3)}$$

$$= \lim_{h \to 0} \frac{9 + h - 9}{h(\sqrt{9+h} + 3)} = \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{9+h} + 3)} = \lim_{h \to 0} \frac{1}{\sqrt{9+h} + 3} = \frac{1}{\sqrt{9} + 3}$$

$$= \frac{1}{3+3} = \boxed{\frac{1}{6}}$$

OR $$f'(9) = \lim_{x \to 9} \frac{f(x) - f(9)}{x - 9} = \lim_{x \to 9} \frac{\sqrt{x} - \sqrt{9}}{x - 9} = \lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 3} \cdot \frac{\sqrt{x} + 3}{\sqrt{x} + 3}$$

$$= \lim_{x \to 9} \frac{(\sqrt{x})^2 - (3)^2}{(x-3)(\sqrt{x} + 3)} = \lim_{x \to 9} \frac{\cancel{(x-3)}}{\cancel{(x-3)}(\sqrt{x} + 3)} = \lim_{x \to 9} \frac{1}{\sqrt{x} + 3}$$

$$= \frac{1}{\sqrt{9} + 3} = \frac{1}{3+3} = \boxed{\frac{1}{6}}$$

(b). Find the equation of the tangent line to $f(x) = \sqrt{x}$ at $x = 9$.

1. pt: $y = \sqrt{9} = 3 \Rightarrow$ pt $(9, 3)$

2. slope: $m = f'(9) = \frac{1}{6}$ from part (a).

$$\boxed{y - 3 = \frac{1}{6}(x - 9)}$$