[Note: The Final Exam is comprehensive. Use the old review sheets, exams, and quizzes to study previous material.]

1. Evaluate the following integrals. [**Note:** You may or may not need to use substitution.] Check your answer by differentiating the result.

(a). $\int_0^2 t^2\sqrt{1+t^3}\,dt = \frac{52}{9}$ — u-substitution

(b). $\int \sin x\cos(\cos x)\,dx = -\sin(\cos x) + C$ — u-substitution

(c). $\int 3x^5 - 4x^3 + 6x + 2\,dx = \frac{1}{2}x^6 - x^4 + 3x^2 + 2x + C$ — direct integration

(d). $\int (3x-1)(3x^2-2x)^2\,dx = \frac{1}{6}(3x^2-2x)^3 + C$ — u-substitution

(e). $\int x(3x^2-2x)^2\,dx = \frac{3}{2}x^6 - \frac{12}{5}x^5 + x^4 + C$ — expand/simplify

(f). $\int \left(1+\frac{1}{t}\right)\left(\frac{1}{t^2}\right)\,dt = -\frac{1}{t} - \frac{1}{2t^2} + C$ — multiply/simplify OR u-substitution

(g). $\int_0^{\pi/6} \sec x\tan x\,dx = \frac{2}{\sqrt{3}} - 1$ — direct integration rule

(h). $\int \sin x\cos x\,dx = \frac{1}{2}\sin^2 x + C$ OR $-\frac{1}{2}\cos^2 x + C$ — u-substitution

(i). $\int \frac{5x}{\sqrt[3]{1-x^2}}\,dx = -\frac{15}{4}(1-x^2)^{2/3} + C$ — u-substitution

(j). $\int_1^3 \frac{x^2+1}{x^2}\,dx = \frac{8}{3}$ — simplify

(k). $\int y^2\sqrt{y}\,dy = \frac{2}{7}y^{7/2} + C$ — simplify

(l). $\int_0^1 (2-x)^6\,dx = \frac{127}{7}$ — u-substitution

(m). $\int \theta\sin(3\theta^2)\,d\theta = -\frac{1}{6}\cos(3\theta^2) + C$ — u-substitution

2. Sketch the region bounded by the graphs of the following functions. Find the ***area*** of the region.

(a). $f(x) = 3 - 2x - x^2, \;\; g(x) = -x + 1$ $\frac{9}{2}$

(b). $x = y^2, \;\; x = -y$ $\frac{1}{6}$

3. Find the ***volume*** of the solid generated by rotating the region bounded by the given curves about the given line.

(a). $y = x^2, y = 4x - x^2$ about the line $y = 6$

$$V = \int_0^2 \pi\left[(6 - x^2)^2 - (6 - 4x + x^2)^2\right] = \frac{64\pi}{3}$$

(b). $xy = 6, y = 2, y = 6, x = 6$ about the line $x = 6$. [**Set up, but do not evaluate!!!**]

$$V = \int_2^6 \pi\left(6 - \frac{6}{y}\right)^2 dy$$

4. The force exerted by gravity on an object sent into space is given by $F(x) = \dfrac{4.8 \times 10^{11}}{x^2}$ pounds where x is measured in miles from the *center* of the earth. How much work is done to propel a satellite module to 800 miles above the earth. Use 4000 miles for the radius of the earth. [Similar problems not requiring a calculator may be on the test.]

$$W = \int_{4000}^{4800} 4.8 \times 10^{11} x^{-2}\, dx = 2 \times 10^7 \text{ mile· lbs} = 1.056 \times 10^{11} \text{ foot· lbs}$$

5. If 18 J of work is required to stretch a spring 40 cm from it's natural length, find the work required to stretch it an additional 30 cm.

$k = \frac{9}{400}$ (if using cm) $k = 225$ (if converted to m) $\Rightarrow W = 37.125$ J

6. Given $f(x) = \dfrac{4x^2 + 4}{x^2}$

(a). Find the average value of $f(x)$ on the interval $[1, 3]$. $f_{ave} = \frac{16}{3}$

(b). Use the Mean Value Theorem for integrals to find all values $x = c$ where $f(c) = f_{ave}$. $x = \sqrt{3}$

7. Find the value of k so that the average value of $f(x) = kx^2 - x$ on [0,2] is equal to 4. $k = \frac{15}{4}$