1. Apply the Mean Value Theorem to the function $f(x) = \sqrt{x-2}$ on the interval $[2, 6]$ and find all values of c that satisfy the MVT. $c = 3$

2. Evaluate the following limits. [Show all work - no shortcuts].

(a). $\lim_{x\to\infty} \frac{3 - x^2 + 4x^3}{x^4 + 2x} = 0$

(b). $\lim_{x\to-\infty} \frac{2x^2 + 3x + 1}{3x^2 - 3x - 4} = \frac{2}{3}$

3. Find $\lim_{x\to\pm\infty} f(x)$ for the following functions and determine any horizontal and slant asymptotes.

(a). $f(x) = \frac{2x+1}{\sqrt{4x^2 - x}}$

(b). $g(x) = \frac{2x^3 - 3x^2 + 2}{x^2 - 3x}$

(a). $\lim_{x\to\infty} f(x) = 1$; $\lim_{x\to-\infty} f(x) = -1$ HA: $y = 1$ and $y = -1$

(b) $\lim_{x\to\infty} g(x) = \infty$; $\lim_{x\to-\infty} g(x) = -\infty$ SA: $y = 2x + 3$

4. Given the following function and its derivatives

$$f(x) = \frac{x}{9 - x^2} \qquad f'(x) = \frac{x^2 + 9}{(9 - x^2)^2} \qquad f''(x) = \frac{2x(x^2 + 27)}{(9 - x^2)^3}$$

Use the Summary of Curve Sketching to determine the relevant information. Sketch the graph of the function. Label any maximum and minimum points and inflection points.

domain: All real numbers, except $x \neq \pm 3$

x-intercept(s): $x = 0$

y-intercept: $y = 0$

vertical asymptote(s): $x = 3, x = -3$

horizontal asymptote(s): $y = 0$

slant asymptote: none

critical numbers: none

intervals where increasing: $(\infty, -3) \bigcup (-3, 3) \bigcup (3, \infty)$

intervals where decreasing: none

coordinates of local max/min(s): none

intervals where concave up: $(\infty, -3) \bigcup (0, 3)$

intervals where concave down: $(-3, 0) \bigcup (3, \infty)$

inflection point(s): $(0, 0)$

Use your calculator to graph the function to check and see if you sketched it correctly.

5. If a box with a square base and open top is to hold 4 ft^3, find the dimensions of the box that will require the least amount of material. $2' \times 2' \times 1'$

6. Find the maximum possible volume of a right circular cylinder if its total surface area (including top and bottom) is 150π.
$V = 250\pi$ $(r = 5, h = 10)$

7. A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window. Find the dimensions of the Norman window with the largest possible area if the total perimeter is 16 ft. Rect. portion: $\frac{16}{\pi+4} \times \frac{32}{\pi+4}$ ft.

8. Given $f(x) = 4x^3 - 12x^2 + 12x - 3$

(a). Explicitly write out Newton's formula for finding the root of this function. $x_{n+1} = x_n - \frac{4x_n^3 - 12x_n^2 + 12x_n - 3}{12x_n^2 - 24x_n + 12}$

(b). Starting with $x_0 = 0.5$, demonstrate Newton's method by marking $x_0, x_1, x_2, \ldots$ and the associated tangent lines on the graph of $f(x)$. Does it seem like Newton's method will work if you start with this initial guess? Yes

(c). Starting with $x_0 = 1.0$, demonstrate Newton's method by marking $x_0, x_1, x_2, \ldots$ and the associated tangent lines on the graph of $f(x)$. Does it seem like Newton's method will work if you start with this initial guess? Why or why not?
No, it won't work since the tangent line is horizontal when $x_0 = 1.0$.

9. Find the antiderivatives for the following functions.

(a). $h(x) = 3x^3 - 7x^2$ $H(x) = \frac{3}{4}x^4 - \frac{7}{3}x^3 + C$

(b). $f(x) = \sqrt{x} - \sqrt{3}x^2$ $F(x) = \frac{2}{3}x^{3/2} - \frac{\sqrt{3}}{3}x^3 + C$

10. Given that $g'(\theta) = -\sec^2\theta$ and $g\left(\frac{\pi}{3}\right) = 0$, find $g(\theta)$. $g(\theta) = -\tan\theta + \sqrt{3}$

11. Given the function $f(x) = \frac{3}{x}$, estimate the area under the curve $f(x)$ on the interval $[1, 6]$ using 5 subintervals and using the right endpoint of each subinterval. [i.e. find R_5].
$R_5 = \Delta x \cdot [f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)] = 1 \cdot [f(2) + f(3) + f(4) + f(5) + f(6)] = \boxed{1 \cdot [3/2 + 3/3 + 3/4 + 3/5 + 3/6]} = \frac{87}{20}$

12. Using the definition of the definite integral $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty} R_n$, ***set-up, but do not evaluate***, the summation/limit using right endpoints for the integral $\int_0^1 x^3 + 1\,dx$. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left[\left(\frac{i}{n}\right)^3 + 1\right]$

13. Evaluate the limit $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left(\frac{i}{n}\right)^2 = \frac{1}{3}$

14. Section 4.3: #3

15. Evaluate the following integrals [Use integration techniques, ***not*** the limit definition.]:

(a). $\int_1^2 t + 2\,dt = \frac{7}{2}$ **(b).** $\int_1^{x^2} t + 2\,dt = \frac{1}{2}x^4 + 2x^2 - \frac{5}{2}$ **(c).** $\int_0^4 \frac{x(2+x)}{\sqrt{x}}\,dx = \frac{352}{15}$

16. Use the Fundamental Theorem of Calculus (Part B/1) to find $F'(x)$

(a). $F(x) = \int_0^x t\cos t\,dt$ $F'(x) = x\cos x$ **(b).** $F(x) = \int_{-2}^{x^2} \sqrt{t+8}\,dt$ $F'(x) = \sqrt{x^2+8} \cdot 2x = 2x\sqrt{x^2+8}$

17. A particle moves with a velocity of $v(t) = -t^2 + 4t$ on the interval $0 \le t \le 6$.

(a). Find the displacement 0 **(b).** Find the total distance traveled $\frac{32}{3} + \left|-\frac{32}{3}\right| = \frac{64}{3}$

18. Let $r(t)$ be the rate at which the world's oil is consumed, where t is measured in years starting at $t = 0$ on January 1, 2000 and $r(t)$ is measured in barrels per year. What does $\int_0^8 r(t)\,dt$ represent and what are its units? The integral represents the change in the amount of oil consumed from 2000 to 2008. The units are are barrels.

19. Evaluate the following integrals. [**Note:** You may or may not need to use substitution.] Check your answer by differentiating the result.

(a). $\int 3x^5 - 4x^3 + 6x + 2\,dx = \frac{1}{2}x^6 - x^4 + 3x^2 + 2x + C$

(b). $\int (3x-1)(3x^2-2x)^2\,dx = \frac{1}{6}(3x^2-2x)^3 + C$

(c). $\int x(3x^2-2x)^2\,dx = \frac{3}{2}x^6 - \frac{12}{5}x^5 - x^4 + C$

(d). $\int \left(1 + \frac{1}{t}\right)\left(\frac{1}{t^2}\right)\,dt = -\frac{1}{t} - \frac{1}{2t^2} + C$

(e). $\int_0^{\pi/6} \sec x\tan x\,dx = \frac{2}{\sqrt{3}} - 1$

(f). $\int \sin x\cos x\,dx = \frac{1}{2}\sin^2 x + C$

(g). $\int \frac{5x}{\sqrt[3]{1-x^2}}\,dx = \frac{15}{4}(1-x^2)^{2/3} + C$

(h). $\int_1^3 \frac{x^2+1}{x^2}\,dx = \frac{8}{3}$

(i). $\int (2-x)^6\,dx = -\frac{(2-x)^7}{7} + C$

(j). $\int \theta\sin(3\theta^2)\,d\theta = -\frac{1}{6}\cos(3\theta^2) + C$