Name: Key

Math 151, Calculus I – Crawford

Exam 3

02 May 2017

Score	
1	/6
2	/8
3	/8
4	/10
5	/14
6	/10
7	/8
8	/24
9	/6
10	/10
Total	/100

- Calculators, books, notes (in any form), cell phones, and any unauthorized sources are ***not*** allowed.
- Clearly indicate your answers.
- ***Show all your work*** – partial credit may be given for written work.
- ***Good luck***!

The following formulas may or may not be helpful.

$$\sum_{i=1}^{n} c = cn \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2 = \frac{n^2(n+1)^2}{4}$$

1. (6 pts). Evaluate the following limit. [Show all algebraic work to justify your answer - no shortcuts.]

$$\lim_{x\to-\infty} \frac{2-3x+x^2}{6x^2-2} = \lim_{x\to-\infty} \frac{x^2\left(\frac{2}{x^2}-\frac{3}{x}+1\right)}{x^2\left(6-\frac{2}{x^2}\right)} = \lim_{x\to-\infty} \frac{\frac{2}{x^2}-\frac{3}{x}+1}{6-\frac{2}{x^2}} = \boxed{\frac{1}{6}}$$

OR $$\lim_{x\to-\infty} \frac{2-3x+x^2}{6x^2-2}\cdot\frac{\left(\frac{1}{x^2}\right)}{\left(\frac{1}{x^2}\right)} = \lim_{x\to-\infty} \frac{\frac{2}{x^2}-\frac{3}{x}+1}{6-\frac{2}{x^2}} = \boxed{\frac{1}{6}}$$

2. (8 pts). Given the equation $2x^3-3x^2-5=0$

(a). Explicitly write out Newton's formula for finding the root of this equation.

$f(x) = 2x^3-3x^2-5$

$f'(x) = 6x^2-6x$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\boxed{x_{n+1} = x_n - \frac{2x_n^3-3x_n^2-5}{6x_n^2-6x_n}}$$

(b). Start with an initial guess of $x_0 = 2$ and iterate Newton's method to find x_1. [Do not simplify.]

$x_0 = 2$

$$x_1 = x_0 - \frac{2x_0^3-3x_0^2-5}{6x_0^2-6x_0}$$

$$\boxed{x_1 = 2 - \frac{2(2)^3-3(2)^2-5}{6(2)^2-6(2)}}$$

3. (8 pts). Determine the slant asymptote of the following function.

$$f(x) = \frac{8x^3 + 2x^2 + 6}{2x^2} = \frac{8x^3}{2x^2} + \frac{2x^2}{2x^2} + \frac{6}{2x^2} = \underbrace{4x+1}_{\text{Slant Asymptote is } \boxed{y = 4x+1}} + \underbrace{\frac{6}{2x^2}}_{0 \text{ as } x \to \pm\infty}$$

OR Long Division:

$$\begin{array}{r} 4x + 1 \\ 2x^2 \overline{)\, 8x^3 + 2x^2 + 0x + 6} \\ -8x^3 \qquad\qquad\qquad \\ \hline 2x^2 \qquad\qquad \\ -2x^2 \qquad\qquad \\ \hline 0 + 0x + 6 \end{array} \leftarrow \text{Remainder}$$

ie $\frac{8x^3+2x^2+6}{2x^2} = 4x+1+\frac{6}{2x^2}$ ← same

4. (10 pts). Given $f(x) = \dfrac{1}{x}$ on the interval $[1, 4]$,

(a). What two conditions on f must be satisfied for the Mean Value Theorem apply?

① $f(\frac{1}{x}) = \frac{1}{x}$ must be continuous on $[1,4]$. ✓

② $f(x) = \frac{1}{x}$ must be differentiable on $(1,4)$. ✓

(b). Find all numbers c that satisfy the conclusion of the Mean Value Theorem.

$f(x) = \frac{1}{x} = x^{-1}$

$f'(x) = -x^{-2} = -\frac{1}{x^2}$

$$\frac{f(b)-f(a)}{b-a} = \frac{f(4)-f(1)}{4-1} = \frac{\frac{1}{4}-\frac{1}{1}}{3} = \frac{\left(\frac{-3}{4}\right)}{\left(\frac{3}{1}\right)} = \frac{-3}{4}\cdot\frac{1}{3} = \frac{-1}{4}$$

Set $f'(x) = \frac{-1}{4}$ and solve

$-\frac{1}{x^2} = -\frac{1}{4}$

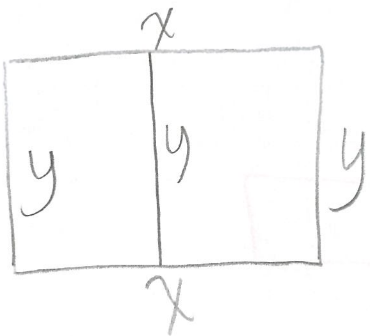
$-x^2 = -4$

$x^2 = 4$

$x = \pm 2$

$\boxed{x = 2 \text{ is only sol}^{\text{n}} \text{ in } [1,4]}$

5. (14 pts). A homeowner ~~wants~~ has 120 feet of fencing to create a rectangular pen for his two dogs. The rectangular area will be divided in half with a fence parallel to one of the sides. What are the dimensions of the overall rectangular region that will yield the maximum area enclosed?



$A = xy \qquad F = 2x + 3y = 120$

Maximize $A = xy$ subject to $2x + 3y = 120$.

$\Rightarrow 3y = 120 - 2x$

$y = \dfrac{120 - 2x}{3}$

$y = 40 - \frac{2}{3}x$

$A = x\left(40 - \frac{2}{3}x\right)$

$A = 40x - \frac{2}{3}x^2$

$A' = 40 - \frac{4}{3}x = 0$

$\frac{4}{3}x = 40$

$x = \overset{10}{\cancel{40}} \cdot \frac{3}{\cancel{4}}$

$x = 30$

Then $y = 40 - \frac{2}{\cancel{3}}(\overset{10}{\cancel{30}})$

$= 40 - 20$

$= 20$

ie $\boxed{30 \text{ ft} \times 20 \text{ ft}}$

6. (10 pts). Using the definition of the definite integral $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$, ***set-up, but do not evaluate***, the summation/limit using right endpoints for the following integral.

$$\int_1^5 x - x^4\,dx$$

$$\Delta x = \frac{b-a}{n} = \frac{5-1}{n} = \frac{4}{n}$$

$$x_i = a + i\Delta x = 1 + i\left(\frac{4}{n}\right) = 1 + \frac{4}{n}i$$

$$f(x_i) = x_i - x_i^4 = \left(1+\frac{4}{n}i\right) - \left(1+\frac{4}{n}i\right)^4$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \boxed{\lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(1+\frac{4}{n}i\right) - \left(1+\frac{4}{n}i\right)^4\right]\cdot\frac{4}{n}}$$

7. (8 pts). Evaluate the following limit.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2}{n}i\right)\cdot\frac{2}{n} = \lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \frac{2}{n}i$$

$$= \lim_{n\to\infty} \frac{4}{n^2} \sum_{i=1}^{n} i$$

$$= \lim_{n\to\infty} \frac{\cancel{4}^{2}}{n^{\cancel{2}}} \cdot \frac{\cancel{n}(n+1)}{\cancel{2}}$$

$$= \lim_{n\to\infty} \frac{2(n+1)}{n} = \lim_{n\to\infty} \frac{2n+2}{n} = \boxed{2}$$

8. (24 pts). Evaluate the following integrals. [Use integration techniques, ***NOT*** the limit definition.]

(a). $\int x^2 - 3x + \frac{4}{x^3}\,dx$ $= \int x^2 - 3x + 4x^{-3}\,dx$

$= \boxed{\frac{1}{3}x^3 - \frac{3}{2}x^2 + 4\frac{x^{-2}}{-2} + C}$

$= \frac{1}{3}x^3 - \frac{3}{2}x^2 - \frac{2}{x^2} + C$

(b). $\int_1^2 (3x+2)(x-1)\,dx$ [Simplify.]

$= \int_1^2 3x^2 - x - 2\,dx$

$= x^3 - \frac{1}{2}x^2 - 2x \Big|_1^2$

$= (2)^3 - \frac{1}{2}(2)^2 - 2(2) - \left[(1)^3 - \frac{1}{2}(1)^2 - 2(1)\right]$

$= 8 - 2 - 4 - \left[1 - \frac{1}{2} - 2\right]$

$= 2 - \left(-\frac{3}{2}\right)$

$= 2 + \frac{3}{2} = \frac{4+3}{2}$

$= \boxed{\frac{7}{2}}$

(c). $\int \underbrace{x^3 \sec^2(x^4)\,dx}_{\frac{1}{4}du}$

$u = x^4$

$du = 4x^3\,dx$

$\frac{1}{4}du = x^3\,dx$

$= \frac{1}{4}\int \sec^2 u\,du$

$= \frac{1}{4}\tan u + C$

$= \boxed{\frac{1}{4}\tan(x^4) + C}$

9. (6 pts). Use the Fundamental Theorem of Calculus Part B/1 to find $F'(x)$ for

$$F(x) = \int_{\pi/4}^{x^3} \cos(t^2)\, dt$$

$$F'(x) = \frac{d}{dx}\left[\int_{\pi/4}^{x^3} \cos(t^2)\, dt\right]$$

$$= \cos((x^3)^2) \cdot 3x^2$$

$$= \boxed{3x^2 \cos(x^6)}$$

10. (10 pts). The velocity (m/sec) of a particle is given by $v(t) = 2t - 4$.

(a). Find the displacement over $0 \le t \le 3$.

$$\int_0^3 2t - 4\, dt = t^2 - 4t \Big|_0^3$$

$$= (3)^2 - 4(3) - (0)$$

$$= 9 - 12 = \boxed{-3}$$

(b). Find the total distance traveled over $0 \le t \le 3$.

$$2t - 4 = 0 \Rightarrow 2t = 4 \Rightarrow t = 2$$ Split up interval at 2.

① $\int_0^2 2t - 4\, dt = t^2 - 4t \Big|_0^2 = (2)^2 - 4(2) = 4 - 8 = -4$ 4 units in neg. direction

② $\int_2^3 2t - 4\, dt = t^2 - 4t \Big|_2^3 = (3)^2 - 4(3) - ((2)^2 - 4(2))$

$= 9 - 12 - (4 - 8)$

$= -3 - (-4) = 1$ 1 unit in the positive direction

Total Distance $= |-4| + |1|$

$= 4 + 1 = \boxed{5}$