Name: Key

Math 151, Calculus I – Crawford

Exam 2

05 April 2016

	Score	
1		/12
2		/20
3		/14
4		/12
5		/10
6		/14
7		/20
Total		/100

- Calculators, books, notes (in any form), cell phones, and any unauthorized sources are ***not*** allowed.
- Clearly indicate your answers.
- ***Show all your work*** – partial credit may be given for written work.
- ***Good luck***!

1. (12 pts). Find the ***absolute*** maximum and minimum ***values*** of $f(x) = 2x^3 - 3x^2 + 5$ on the interval $[-1, 1]$.

$f'(x) = 6x^2 - 6x = 0$

$6x(x-1) = 0$

$6x = 0$ or $x - 1 = 0$

$x = 0, 1$

$f(0) = 5 \leftarrow$ $\boxed{\text{Abs max. is } 5}$ at $x = 0$

$f(1) = 2(1)^3 - 3(1)^2 + 5 = 2 - 3 + 5 = 4$

endpts $-1, 1$ ↑ Already done

$f(-1) = 2(-1)^3 - 3(-1)^2 + 5 = -2 - 3 + 5 = 0 \leftarrow$ $\boxed{\text{Abs min. is } 0}$ at $x = -1$

2. (20 pts). Differentiation. **[Do not simplify!]**

(a). Differentiate: $y = \dfrac{1}{\sqrt{2x^6 - 3x + 1}}$ $= (2x^6-3x+1)^{-1/2}$

$$\boxed{y' = -\tfrac{1}{2}(2x^6-3x+1)^{-3/2} \cdot (12x^5-3)}$$

OR $y = \dfrac{1}{(2x^6-3x+1)^{1/2}}$

$$y' = \frac{(2x^6-3x+1)^{1/2}\cdot 0 - 1\cdot \frac{1}{2}(2x^6-3x+1)^{-1/2}\cdot(12x^5-3)}{\left((2x^6-3x+1)^{1/2}\right)^2}$$

(b). Find the first **AND** second derivatives of $f(\theta) = \sec(5\theta)$. CR

$$\boxed{f'(\theta) = \sec(5\theta)\tan(5\theta)\cdot 5}$$

Simplified: $f'(\theta) = \underbrace{5\sec(5\theta)}_{1^{st}}\underbrace{\tan(5\theta)}_{2^{nd}}$

CR CR

$$\boxed{f''(\theta) = 5\sec(5\theta)\cdot\sec^2(5\theta)\cdot 5 + \tan(5\theta)\cdot 5\sec(5\theta)\tan(5\theta)\cdot 5}$$

3. (14 pts). Given the curve $y \cos x = x^2 + 4x + y^3$.

(a). Use implicit differentiation to find $\frac{dy}{dx}$.

$$\frac{d}{dx}\left[y\cos x\right] = \frac{d}{dx}\left[x^2+4x+y^3\right]$$

$$y\cdot(-\sin x) + (\cos x)y' = 2x+4+3y^2y'$$

$$(\cos x)y' - 3y^2y' = 2x+4+y\sin x$$

$$y'(\cos x - 3y^2) = 2x+4+y\sin x$$

$$\boxed{y' = \frac{2x+4+y\sin x}{\cos x - 3y^2}}$$

(b). Find the equation of the tangent line to the curve at $(0, 1)$.

① pt ✓ $(0,1)$

② slope: $y'\big|_{(0,1)} = \frac{2(0)^{\to 0}+4+(1)\sin(0)^{\to 0}}{\cos(0) - 3(1)^2} = \frac{4}{1-3}$

$$= \frac{4}{-2} = -2 = m$$

$$\boxed{y - 1 = -2(x-0)}$$

$$y = -2x+1$$

4. (12 pts). Given $f(x) = \dfrac{1}{(1+x)^3} = (1+x)^{-3}$

$$\boxed{\begin{aligned} L(x) &= 1 - 3(x-0) \\ L(x) &= 1 - 3x \end{aligned}}$$

(a). Find the linearization $L(x)$ at $a = 0$

① pt: $f(0) = \dfrac{1}{(1+0)^3} = \dfrac{1}{1^3} = 1$

② slope: $f'(x) = -3(1+x)^{-4} = \dfrac{-3}{(1+x)^4}$

$f'(0) = \dfrac{-3}{(1+0)^4} = \dfrac{-3}{1^4} = -3 = m$

$L(x) = f(0) + f'(0)(x-0)$

OR

$y - 1 = -3(x-0)$

$y - 1 = -3x$

$y = 1 - 3x$

$L(x) = 1 - 3x$

(b). Use this linearization $L(x)$ to approximate $f(0.2)$. Simplify your answer.

$f(0.2) \approx L(0.2) = 1 - 3(0.2)$

$= 1 - 0.6$

$= \boxed{0.4}$

5. (10 pts). Solve the following equation for **all** values of x.

$\sin(x)\cos(3x) + \sin x = 0$

$\sin x(\cos(3x) + 1) = 0$

$\sin x = 0$ or $\cos(3x) + 1 = 0$

$\cos(3x) = -1$

$3x = \pi + 2\pi n$

$x = 0 + 2\pi n$

or $x = \pi + 2\pi n$

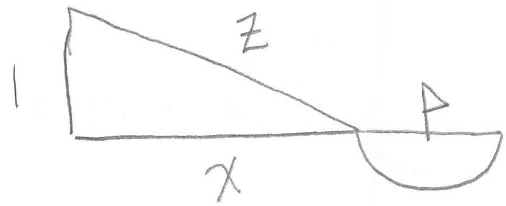
$x = \dfrac{\pi}{3} + \dfrac{2\pi n}{3}$

$n = 0, \pm 1, \pm 2, \ldots$

6. (14 pts). A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 1 m higher than the bow of the boat. If the rope is pulled in at a rate of 2 m/s, how fast is the boat approaching the dock when it is 4 m from the dock?

[Remember that significant partial credit will be given for clearly and accurately labeling the picture, and indicating values and equations in correct mathematical notation.]

Step 1



Step 2

X = Dist. from dock $\frac{dx}{dt}$ = Vel. of boat (i.e. rate it moves)

Z = Length of Rope $\frac{dz}{dt}$ = rate rope is pulled in

Step 3

Find $\frac{dx}{dt}$ when $X = 4$

$\frac{dz}{dt} = -2$

Step 4

$X^2 + 1^2 = Z^2$

$X^2 + 1 = Z^2$

Step 5

$\frac{d}{dt}[X^2+1] = \frac{d}{dt}[Z^2]$

$\cancel{2}X \cdot \frac{dx}{dt} = \cancel{2}Z \cdot \frac{dz}{dt}$

$X \frac{dx}{dt} = Z \frac{dz}{dt}$

$\frac{dx}{dt} = \frac{Z}{X}\frac{dz}{dt}$

still need Z : at same instant.

$X^2 + 1 = Z^2$

$(4)^2 + 1 = Z^2$

$17 = Z^2$

$Z = \pm\sqrt{17}$

$Z = \sqrt{17}$

[Include units in your answer.]

Step 6

$\frac{dx}{dt} = \frac{\sqrt{17}}{\cancel{4}_{2}}(-\cancel{2}) = \boxed{\frac{-\sqrt{17}}{2} \text{ m/s}}$

7. (20 pts). Given the following function and its derivatives.

$$f(x) = \frac{x^2}{x+2} \qquad f'(x) = \frac{x(x+4)}{(x+2)^2} \qquad f''(x) = \frac{8}{(x+2)^3}$$

(a). Find the intervals on which f is increasing or decreasing.

$\underline{f' = 0}$

$x(x+4) = 0$

$x = 0, -4$

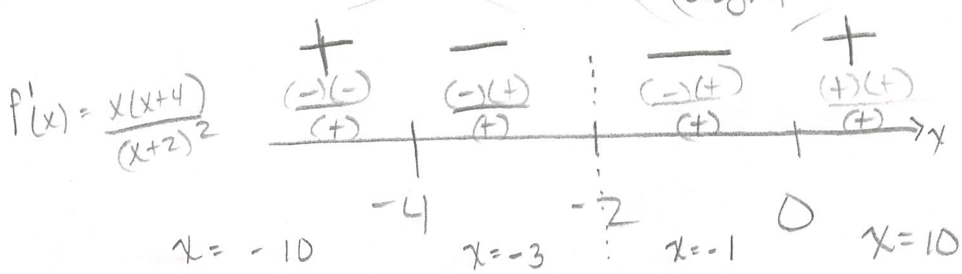
$\underline{f' \text{ DNE}}$

$(x+2)^2 = 0$

$x = -2$

(asymptote in f.)

$$\boxed{\begin{array}{l}\text{increasing on} \\ (-\infty,-4)\cup(0,\infty) \\ \text{decreasing on} \\ (-4,-2)\cup(-2,0)\end{array}}$$



(b). Find the local maximum and minimum ***values***.

Local max at $x=-4$: $f(-4) = \frac{(-4)^2}{-4+2} = \frac{16}{-2} = \boxed{-8}$ Local max value

Local min at $x=0$: $f(0) = \frac{0^2}{0+2} = \frac{0}{2} = \boxed{0}$ local min Value

(c). Find the intervals of concavity.

$\underline{f'' = 0}$

$8 = 0$

Not possible

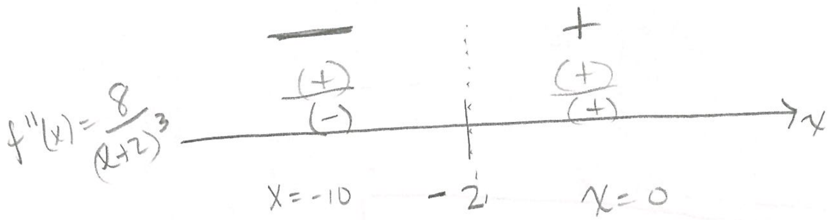
$\underline{f'' \text{ DNE}}$

$(x+2)^3 = 0$

$x = -2$

(asymptote inf)

$$\boxed{\begin{array}{l}\text{Concave up on} \\ (-2,\infty) \\ \text{Concave Down on} \\ (-\infty,-2)\end{array}}$$



(d). Find the inflection ***points***.

$\boxed{\text{No Inflection Pts.}}$ (x = -2 is asymptote)