1. Find the domain and sketch the function

(a). $f(x) = \begin{cases} -1, & x \le -1 \\ x, & -1 < x \le 2 \\ x^2+1, & x > 2 \end{cases}$ domain: All real numbers

(b). $f(x) = \sqrt{x+4}$ domain: $x \ge -4$

2. Determine whether the $f(x) = x^{-3} + x$ is odd, even, or neither. odd

3. Graph $y = \frac{1}{2}\tan\left(x + \frac{\pi}{3}\right)$

4. Solve the following inequality for x: $2x^2 + x \ge 3$ $(-\infty, -\frac{3}{2}] \bigcup [1, \infty)$

5. Given $f(x) = \frac{1}{x} - 3$ and $g(x) = \sqrt{x+3}$, find the following composite functions and state their domains explicitly for x.

(a). $f(g(x)) = \dfrac{1}{\sqrt{x+3}} - 3$ Domain: $x > -3$

(b). $g \circ f = g(f(x)) = \sqrt{\dfrac{1}{x}}$ Domain: $x > 0$

6. Section 1.3 #3 a: 3 b: 1 c: 4 d: 5 e: 2

7. Evaluate the following limits, if they exist (clearly indicate $+\infty$ or $-\infty$ in the case of an infinite limit). <u>If the limit does not exist, **explain the reason why**.</u>

(a). $\lim_{x \to 0} \dfrac{x-3}{x(x+4)}$ DNE (one-sided limits are different)

(b). $\lim_{x \to -4} \dfrac{x^2+2x-8}{x+4} = -6$

(c). $\lim_{x \to 0} \dfrac{\sqrt{3x^2+4}}{x-4} = -\dfrac{1}{2}$

(d). $\lim_{x \to 0} \dfrac{x-3}{x^2(x+4)} = -\infty$

(e). $\lim_{h \to 0} \dfrac{\sqrt{x+h} - \sqrt{x}}{h} = \dfrac{1}{2\sqrt{x}}$

(f). $\lim_{x \to 1} f(x)$, where $f(x) = \begin{cases} 3, & x \le 1 \\ 1, & x > 1 \end{cases}$
DNE (one-sided limits are different)

8. Given that $\lim_{x \to 1} f(x) = 2$, $\lim_{x \to 1} g(x) = 4$, $\lim_{x \to 1} h(x) = 0$, find the following limits if they exist.

(a). $\lim_{x \to 1} f(x) - g(x) = -2$

(b). $\lim_{x \to 1} f(x) \cdot g(x) = 8$

(c). $\lim_{x \to 1} h(x)/g(x) = 0$

(d). $\lim_{x \to 1} g(x)/h(x)$ DNE (One-sided limits are $+\infty$ or $-\infty$, but not enough info to determine which one or if they agree.)

9. Given the function $f(x) = x^3 - 2x^2 + 8x - 1$, use the Intermediate Value Theorem to show that there is a number c where $0 < c < 2$, such that $f(c) = 6$. $f(0) = -1$ and $f(2) = 15$. Since $-1 \le 6 \le 15$ AND f is continuous, then the IVT guarantees $f(x)$ must pass through $y = 6$ for some value of $x = c$ in the interval $(0, 2)$.

10. For each of the following functions,
(i) find all of the x-values, if any, where g(x) is discontinuous and
(ii) indicate whether it is a removable, infinite, or jump discontinuity.

(a). $g(x) = \dfrac{x^2-x-6}{x(x^2-9)}$ infinite at $x = 0$, infinite $x = -3$, removable at $x = 3$

(b). $f(x) = \begin{cases} x^2-1, & x \le 1 \\ 1-x & x > 1 \end{cases}$ Continuous everywhere

11. Suppose $f(1) = 3, f'(1) = -2, f(5) = 8$, and $f'(5) = 15$. Let P be the point on the graph $y = f(x)$ where $x = 1$. Let Q be the point on the graph of $y = f(x)$ where $x = 5$.

(a). Find the equation of the secant line PQ. $y - 3 = \frac{5}{4}(x - 1)$

(b). Find the equation of the tangent line to $y = f(x)$ at P. $y - 3 = -2(x - 1)$

12. Suppose the position of a particle at time t seconds is given by $s(t) = \sqrt{t}$ meters. ***Use the limit definition*** of the derivative to find the velocity of the particle at time $t = 5$. $v(5) = \dfrac{1}{2\sqrt{5}}$

13. ***Use the limit definition*** of the derivative to find $f'(x)$ for the following: [You **must** use the limit definition.]

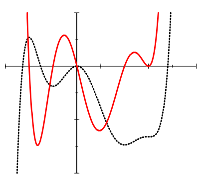
(a). $f(x) = \dfrac{1}{x^2}$ ***Simplify your answer.*** $f'(x) = -\dfrac{2}{x^3}$

(b). $f(x) = 2x^2 + 3x$ $f'(x) = 4x + 3$

14. Find the equation of the tangent line to the curve $y = 2x^2 + 3x$ at the point (1,5). $y - 5 = 7(x - 1)$
[Hint: See (b) in previous problem.]

15. Section 2.1 #49 $T'(8)$ represents the *rate* at which the temperature is changing at 8am. $T'(8) \approx 3.75°$ F/hour.

16. Both the function f and its derivative f' are plotted on the same set of axes below. Which curve represents the function and which curve represents the derivative? ***Justify your answer.*** dotted is $f(x)$; solid is $f'(x)$



17. Section 2.2 #3, 15, 39.

18. Differentiate the following using ***Differentiation Rules*** [i.e Do ***NOT*** use the limit definition] [Do not simplify!]

(a). $y = 10x^3 - 3x + 7$ $y' = 30x^2 - 3$

(b). $f(x) = \pi^2$ $f'(x) = 0$

(c). $y = (3x)^3$ $\dfrac{dy}{dx} = 81x^2$

(d). $y = \dfrac{x + 4x^3 - 3}{x^3}$ $y' = \dfrac{x^3(1 + 12x^2) - (x + 4x^3 - 3)(3x^2)}{x^6}$
or $y' = -2x^{-3} + 9x^{-4}$

(e). $s(t) = t^2(3t - 4t^3)$ $s'(t) = 9t^2 - 20t^4$

(f). $f(x) = \dfrac{3}{x^2} - \sqrt{x}$ $f'(x) = -6x^{-3} - \dfrac{1}{2}x^{-1/2}$

(g). $y = \dfrac{2x^2(3x^2 + 5)}{x^3 + 2x - 1}$
$y' = \dfrac{(x^3 + 2x - 1)(24x^3 + 20x) - (6x^4 + 10x^2)(3x^2 + 2)}{(x^3 + 2x - 1)^2}$

19. Find the the second derivative of $s(t) = t^2(3t - 4t^3)$. [Note: You already found $s'(t)$ in the previous problem.] $s''(t) = 18t - 80t^3$

20. Find the equation of the tangent line to the curve $y = 10x^3 - 3x + 7$ at $x = -1$.
[Note: This is the same function as part (a) of a previous problem.] $y = 27(x + 1) = 27x + 27$