Name: Key

Math 151, Calculus I – Crawford

Exam 1

28 February 2017

Score	
1	/6
2	/8
3	/6
4	/18
5	/16
6	/14
7	/7
8	/14
9	/7
10	/4
11	/4
Total	/104

- Calculators, books, notes (in any form), cell phones, and any unauthorized sources are ***not*** allowed.
- Clearly indicate your answers.
- ***Show all your work*** – partial credit may be given for written work.
- Problems #4, 5, & 9 will be used to determine extra-credit for Quiz 1.
- ***Good luck***!

1. (6 pts). Find the domain of $f(x) = \dfrac{x+1}{\sqrt{3x+8}}$

$3x + 8 > 0$

$3x > -8$

$\boxed{x > -\frac{8}{3}}$

2. (8 pts). Solve the following inequality for x.

$$\frac{(x-1)^2(x-4)}{x+3} \leq 0$$

Equals 0

$(x-1)^2(x-4)=0$

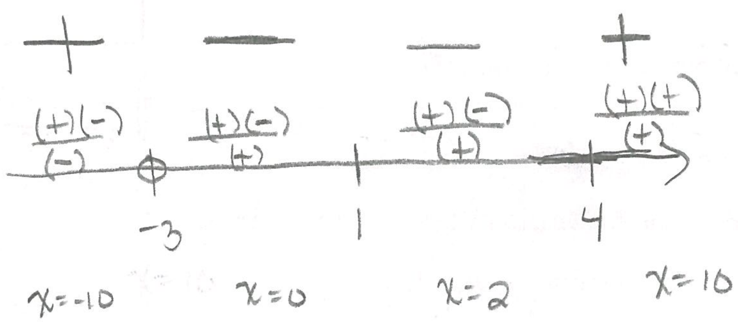
$x = 1, 4$

undefined

$x+3=0$

$x=-3$

$\boxed{\begin{matrix}\underbrace{(-3,1] \cup [1,4]} \\ (-3,4]\end{matrix}}$



3. (6 pts). Given $f(x) = x^2 + 1$ and $g(x) = \dfrac{1}{3-4x}$, find and simplify $g \circ f$.

$$(g \circ f)(x) = g(f(x))$$
$$= g(x^2+1)$$
$$= \frac{1}{3-4(x^2+1)}$$
$$= \frac{1}{3-4x^2-4}$$
$$= \boxed{\frac{1}{-1-4x^2}}$$

4. (18 pts). Evaluate the following limits, if they exist. Clearly indicate $+\infty$ or $-\infty$ in the case of an infinite limit. If the limit does not exist, **clearly explain the reason why**.

(a). $\lim_{x\to 3} \dfrac{x^2-9}{x^2+x-12}$ $= \lim_{x\to 3} \dfrac{(x+3)\cancel{(x-3)}}{(x+4)\cancel{(x-3)}}$

$\dfrac{3^2-9}{3^2+3-12} \to \dfrac{0}{0}$

Ind. Form.

$\Rightarrow$ More Work

$= \lim_{x\to 3} \dfrac{x+3}{x+4}$

$= \dfrac{3+3}{3+4} = \boxed{\dfrac{6}{7}}$

(b). $\lim_{x\to 2} \dfrac{2x-5}{x(x-2)}$

$\dfrac{4-5}{2(0)} \to \dfrac{-1}{0}$

Infinite Limit

$\Rightarrow$ Check one-sided limits

$\lim_{x\to 2^-} \dfrac{2x-5}{x(x-2)} = +\infty$

eg 1.9

$\dfrac{(-)}{(+)(-)} \to (+)$

$\lim_{x\to 2^+} \dfrac{2x-5}{x(x-2)} = -\infty$

eg 2.1

$\dfrac{(-)}{(+)(+)} \to (-)$

So $\lim_{x\to 2} \dfrac{2x-5}{x(x-2)}$ DNE since the one-sided limits do not agree

(c). $\lim_{x\to 4} \dfrac{2-\sqrt{x}}{4-x}$ $= \lim_{x\to 4} \dfrac{2-\sqrt{x}}{4-x} \cdot \dfrac{2+\sqrt{x}}{2+\sqrt{x}}$

$\dfrac{2-\sqrt{4}}{4-4} = \dfrac{0}{0}$

Ind. Form.

$\Rightarrow$ More Work

$= \lim_{x\to 4} \dfrac{(2)^2-(\sqrt{x})^2}{(4-x)(2+\sqrt{x})}$

$= \lim_{x\to 4} \dfrac{\cancel{(4-x)}}{\cancel{(4-x)}(2+\sqrt{x})}$

$= \lim_{x\to 4} \dfrac{1}{2+\sqrt{x}}$

$= \dfrac{1}{2+\sqrt{4}}$

$= \dfrac{1}{2+2}$

$= \boxed{\dfrac{1}{4}}$

5. (16 pts). Let $f(x) = \frac{1}{x}$.

(a). Find the ***slope*** of the secant line connecting the points at $x = 1$ and $x = 4$.

$x=1:\ f(1) = \frac{1}{1} = 1 \Rightarrow P(1,1)$

$x=4:\ f(4) = \frac{1}{4} \Rightarrow Q(4,\frac{1}{4})$

$m_{PQ} = \dfrac{\frac{1}{4}-1}{4-1} = \dfrac{\left(-\frac{3}{4}\right)}{3}$

$= -\frac{3}{4}\cdot\frac{1}{3} = \boxed{-\frac{1}{4}}$

(b). ***Use the limit definition*** $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ to show that the derivative is $f'(x) = -\frac{1}{x^2}$. ***You must show all your work.***

$f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \dfrac{\left(\frac{1}{x+h} - \frac{1}{x}\right)}{h}$

$= \lim_{h\to 0} \dfrac{\left(\frac{x-(x+h)}{x(x+h)}\right)}{h}$

$= \lim_{h\to 0} \dfrac{x-x-h}{x(x+h)} \cdot \dfrac{1}{h}$

$= \lim_{h\to 0} \dfrac{-\cancel{h}}{x(x+h)\cdot\cancel{h}}$

$= \lim_{h\to 0} \dfrac{-1}{x(x+h)}$

$= \dfrac{-1}{x(x)} = \boxed{\dfrac{-1}{x^2}}$ ✓

(c). Find an equation of the tangent line to the graph at $x = 4$.

① pt. $y = f(4) = \frac{1}{4} \Rightarrow (4,\frac{1}{4})$

② slope: $m = f'(4) = \frac{-1}{(4)^2} = \frac{-1}{16}$

$\boxed{y - \frac{1}{4} = -\frac{1}{16}(x-4)}$

For the remainder of the test, use the DIFFERENTIATION RULES to find any needed derivatives.
Do ***NOT*** use the limit definition.

6. (14 pts). Differentiate the following using Differentiation Rules. Do ***NOT*** use the limit definition!
Do not simplify.

(a). $y = 3x^5 - \frac{2}{3}x^4 + \frac{3}{x^5} - \sqrt{x^3}$ $= 3x^5 - \frac{2}{3}x^4 + 3x^{-5} - x^{\frac{3}{2}}$

$$\boxed{\frac{dy}{dx} = 15x^4 - \frac{8}{3}x^3 - 15x^{-6} - \frac{3}{2}x^{\frac{1}{2}}}$$

(b). $g(t) = \frac{(2t+3)(t^2-4t)}{5t^3 - t + 3}$ $= \frac{2t^3 - 8t^2 + 3t^2 - 12t}{5t^3 - t + 3} = \frac{2t^3 - 5t^2 - 12t}{5t^3 - t + 3}$

$$\boxed{g'(t) = \frac{(5t^3 - t + 3)(6t^2 - 10t - 12) - (2t^3 - 5t^2 - 12t)(15t^2 - 1)}{(5t^3 - t + 3)^2}}$$

OR $g'(t) = \frac{(5t^3 - t + 3)\cdot\frac{d}{dt}[(2t+3)(t^2-4t)] - (2t+3)(t^2-4t)\cdot\frac{d}{dt}[5t^3 - t + 3]}{(5t^3 - t + 3)^2}$

$$= \boxed{\frac{(5t^3 - t + 3)[(2t+3)(2t-4) + (t^2-4t)(2)] - (2t+3)(t^2-4t)\cdot(15t^2-1)}{(5t^3 - t + 3)^2}}$$

7. (7 pts). Given $h(x) = x^2 \cdot g(x)$ and $g(2) = 3$ and $g'(2) = -2$, find $h'(2)$

$h'(x) = x^2 \cdot g'(x) + g(x) \cdot 2x$ ← Product Rule

$h'(2) = (2)^2 \cdot g'(2) + g(2) \cdot 2(2)$

$= 4 \cdot (-2) + 3 \cdot 4$

$= -8 + 12$

$= \boxed{4}$

8. (14 pts). Given $y = 3x^2 + 7x - 10$.

(a). Find the equation of the tangent line to y at $(2, 16)$.

① pt ✓ (2,16)

② slope: $y' = 6x + 7 \Big|_{x=2} = 6(2) + 7 = 12 + 7 = 19 = m$

$$\boxed{y - 16 = 19(x - 2)}$$

(b). Find the ***point(s)*** on the curve where the tangent line has a slope of 1.

Slope = $6x + 7 = 1$

$6x = -6$

$x = -1$

pt. on curve ⇒

$y = 3(-1)^2 + 7(-1) - 10$

$= 3 - 7 - 10$

$= -14$

ie pt. $\boxed{(-1, -14)}$

9. (7 pts). Find the value of c so that the following function is continuous for all x.

$$g(x) = \begin{cases} 2x^2 - 1, & x \leq 2 \\ 4 - cx, & x > 2 \end{cases}$$

$g(2) = 2(2)^2 - 1$

$= 8 - 1$

$= 7$

$\lim_{x \to 2^+} g(x) = 4 - c(2)$

$= 4 - 2c$

must be equal for continuity

ie

$4 - 2c = 7$

$-2c = 3$

$$\boxed{c = -\frac{3}{2}}$$

10. (4 pts). *True or False.* Clearly indicate whether the following statements are true or false.

T F If $f(2) = -5$ and $f(10) = 8$, then there must be a number c in the interval $(2, 10)$ such that $f(c) = 0$.

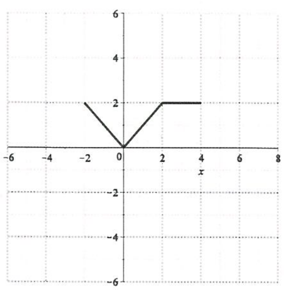
↳ Don't know if f is continuous on [2, 10], so there is no guarantee.

T F The function $f(x) = \dfrac{x}{x^2+1}$ is odd.

$$f(-x) = \frac{-x}{(-x)^2+1} = \frac{-x}{x^2+1} = -f(x)$$

ie $f(-x) = -f(x) \Rightarrow$ ODD

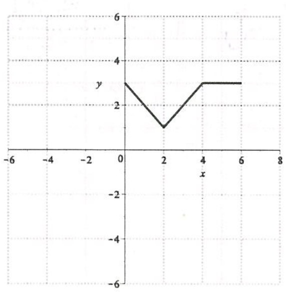
11. (4 pts). Given the graph of $f(x)$ below,



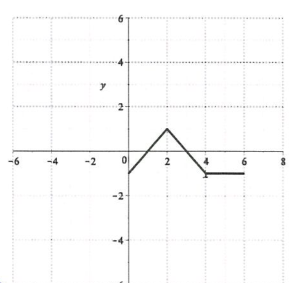
• Shift right by 2
• Reflect through x-axis
• Shift up by 1

(a). Which of the following is a graph of $y = -f(x-2)+1$?

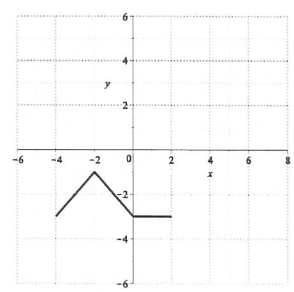
(i)



(ii)

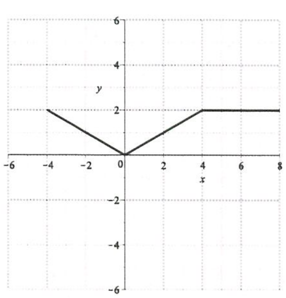


(iii)

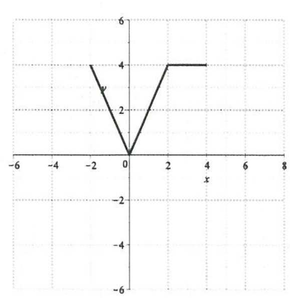


(b). Which of the following is a graph of $y = f(2x)$? Compress horizontally by 2

(i)



(ii)



(iii)