

Recall for  $y = f(x)$ ,

- The average rate of change of  $y$  over the interval  $[x_1, x_2]$  is
- The (instantaneous) rate of change of  $y$  with respect to  $x$  is

Applications of the derivative show up in many branches of science.

---

Density of a thin rod.

If a rod has a uniform mass distribution, then the \_\_\_\_\_ density  $\rho =$

If the rod is not uniform, but rather the mass distribution is a function of position  $x$  (measured from the left end)

i.e., mass

then the average density  $\bar{\rho} =$

and the linear density at position  $x$  is  $\rho =$

**Ex** The mass at position  $x$  of a long, thin rod is given by  $m = \sqrt{x^2 + 1}$  kg, where  $x$  is measured in meters. Find the linear density when  $x = 2$  m.

**Ex** A particle moves according to a law of motion  $s(t) = t^4 - 4t^3 + 10, t > 0$ , where  $t$  is measured in seconds and  $s$  in feet.

- (a). Find the velocity at time  $t$ .
- (b). What is the velocity after 1 second?
- (c). When is the particle at rest?
- (d). When is the particle moving in the positive direction?
- (e). Find the total distance traveled during the first 6 seconds.
- (f). Draw a diagram to illustrate the motion of the particle.
- (g). Find the acceleration at time  $t$  and after 1 second.
- (h). Graph the position, velocity, and acceleration for  $0 \leq t \leq 5$
- (i). When is the particle speeding up? When is it slowing down?

**Ex** Sodium Chlorate crystals can be grown into the shape of a cube by allowing a solution of water and sodium chlorate to evaporate slowly. Let  $V$  be the volume of the cube with side  $x$ .

(a). Find  $\frac{dV}{dx}$  when  $x = 3$  mm and interpret its meaning.

(b). Show that the rate of change of the volume with respect to its edge length is equal to half of the surface area.

**Economics** Let  $C(x)$  be the total cost to produce  $x$  units.

The average rate of change of cost is

The marginal cost is

and represents the approximate cost of producing one more unit.

[Note: Marginal profit and marginal revenue are defined similarly]

**Ex** The profit (in dollars) from selling  $x$  units is given by  $P(x) = .0002x^3 + 10x$ .

(a). Find the marginal profit.

(b). Find the marginal profit if 50 units are currently being sold.

(c). Compare the marginal profit for 50 units with the profit from the 51st unit.

See book for other applications in physics, biology, chemistry, etc.