$\dfrac{d}{dx}[c] = 0$ $\dfrac{d}{dx}[cx] = c$ $\dfrac{d}{dx}[c \cdot f(x)] = c \cdot \dfrac{d}{dx}[f(x)]$

$\dfrac{d}{dx}[x] = 1$ $\dfrac{d}{dx}[x^n] = nx^{n-1}$ $\dfrac{d}{dx}[f(x) \pm g(x)] = \dfrac{d}{dx}[f(x)] \pm \dfrac{d}{dx}[g(x)]$

ALL RULES MUST COME FROM THE LIMIT DEFINITION OF THE DERIVATIVE!

Fill in the spaces/blanks below to prove each of the rules.

1. $f(x) = c$

Then $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \dfrac{\quad\quad}{h} = \lim_{h\to 0} \dfrac{\quad}{h} = \lim_{h\to 0} 0 = 0 \quad \Longrightarrow \quad \boxed{\dfrac{d}{dx}[c] = \quad}$

2. $f(x) = x$

Then $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \dfrac{\quad\quad\quad}{h} = \lim_{h\to 0} \dfrac{h}{h} = \lim_{h\to 0} \quad\quad = 1 \quad \Longrightarrow \quad \boxed{\dfrac{d}{dx}[x] = \quad}$

3. $f(x) = cx$

Then $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \dfrac{\quad\quad - cx}{h} = \lim_{h\to 0} \dfrac{\quad\quad - cx}{h} = \lim_{h\to 0} \dfrac{ch}{h} = \lim_{h\to 0} c = c \quad \Longrightarrow \quad \boxed{\dfrac{d}{dx}[cx] = \quad}$

4. $f(x) = x^n$ [See back page for proof]

5. $F(x) = cf(x)$

Then $F'(x) = \lim_{h\to 0} \dfrac{F(x+h) - F(x)}{h}$

$= \lim_{h\to 0} \dfrac{\quad\quad - cf(x)}{h} = c \cdot \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = c \cdot \underline{\quad\quad\quad} \quad \Longrightarrow \quad \boxed{\dfrac{d}{dx}[cf(x)] = \quad}$

6. $F(x) = f(x) + g(x)$ [Similar proof for $f(x) - g(x)$]

Then $F'(x) = \lim_{h\to 0} \dfrac{F(x+h) - F(x)}{h} = \lim_{h\to 0} \dfrac{\quad\quad\quad - (f(x) + g(x))}{h}$

$= \lim_{h\to 0} \dfrac{f(x+h) + g(x+h) - f(x) - g(x)}{h} = \lim_{h\to 0} \dfrac{f(x+h) - f(x) + \quad\quad\quad}{h}$

$= \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h} + \lim_{h\to 0} \dfrac{g(x+h) - g(x)}{h} = \quad\quad + g'(x) \Longrightarrow \boxed{\dfrac{d}{dx}[f(x) + g(x)] = \quad}$

4. $f(x) = x^n$, for n a positive integer.

Multiply the following polynomials to verify a property that we will use later:

$$(x-a)(x^{n-1} + x^{n-2}a + x^{n-3}a^2 + x^{n-4}a^3 + \ldots + x^2a^{n-3} + xa^{n-2} + a^{n-1})$$

$$= x \cdot (x^{n-1} + x^{n-2}a + x^{n-3}a^2 + x^{n-4}a^3 + \ldots + x^2a^{n-3} + xa^{n-2} + a^{n-1})$$
$$-a \cdot (x^{n-1} + x^{n-2}a + x^{n-3}a^2 + x^{n-4}a^3 + \ldots + x^2a^{n-3} + xa^{n-2} + a^{n-1})$$

$$=$$
$$-(\qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad)$$

$$= x^n + x^{n-1}a + x^{n-2}a^2 + x^{n-3}a^3 + \ldots + x^3a^{n-3} + x^2a^{n-2} + xa^{n-1}$$
$$-\left(x^{n-1}a + x^{n-2}a^2 + x^{n-3}a^3 + x^{n-4}a^4 + \ldots + x^2a^{n-2} + xa^{n-1} + a^n\right)$$

$$=$$

In other words, $x^n - a^n$ factors as $(x-a)(x^{n-1} + x^{n-2}a + x^{n-3}a^2 + \ldots + x^2a^{n-3} + xa^{n-2} + a^{n-1})$

$f(x) = x^n$, for n a positive integer. Find the derivative at $x = a$.

Then $f'(a) = \lim\limits_{x \to a} \dfrac{f(x) - f(a)}{x - a} = \lim\limits_{x \to a} \dfrac{x^n - a^n}{x - a}$

$$= \lim_{x \to a} \frac{\qquad \qquad \qquad \qquad \qquad \qquad}{x - a}$$

$$= \lim_{x \to a} x^{n-1} + x^{n-2}a + x^{n-3}a^2 + \ldots + x^2a^{n-3} + xa^{n-2} + a^{n-1}$$ Note: n terms

$$=$$ Note: n terms

$$= a^{n-1} + a^{n-1} + a^{n-1} + \ldots + a^{n-1} + a^{n-1} + a^{n-1}$$ Note: n terms

$$= \_\_\_\_\_\_\_\_ \cdot a^{n-1}$$ Hint: How many a^{n-1}'s do you have?

Replace a with x and we get $f'(x) = n \cdot x^{n-1}$ $\implies$ $\boxed{\dfrac{d}{dx}\left[x^n\right] = \qquad}$

5. $F(x) = f(x) \cdot g(x)$

Then

$$\begin{aligned}
F'(x) &= \lim_{h\to 0} \frac{F(x+h) - F(x)}{h} \\
&= \lim_{h\to 0} \frac{\qquad\qquad\qquad\qquad - f(x)\cdot g(x)}{h} \\
&= \lim_{h\to 0} \frac{f(x+h)\cdot g(x+h) \qquad\qquad\qquad\qquad\qquad\qquad - f(x)\cdot g(x)}{h} \\
&= \lim_{h\to 0} \frac{[f(x+h)\cdot g(x+h) - f(x+h)\cdot g(x)] + [f(x+h)\cdot g(x) - f(x)\cdot g(x)]}{h} \\
&= \lim_{h\to 0} \frac{f(x+h)\cdot [\qquad\qquad\qquad\qquad] + g(x)\cdot [\qquad\qquad\qquad\qquad]}{h} \\
&= \lim_{h\to 0} \frac{f(x+h)\cdot [g(x+h) - g(x)]}{h} + \frac{g(x)\cdot [f(x+h) - f(x)]}{h} \\
&= \lim_{h\to 0} f(x+h)\cdot \frac{g(x+h) - g(x)}{h} + \qquad\qquad g(x)\cdot \frac{f(x+h) - f(x)}{h}
\end{aligned}$$

[See book for proof of the Quotient Rule.]