Basic Limits

1. $\lim_{x \to a} c = c$

2. $\lim_{x \to a} x = a$

Limit Laws Suppose $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist and c is a constant, then

3. $\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$

4. $\lim_{x \to a} cf(x) = c \lim_{x \to a} f(x)$

5. $\lim_{x \to a} [f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

6. $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$, if $\lim_{x \to a} g(x) \neq 0$

7. $\lim_{x \to a} [f(x)]^n = [\lim_{x \to a} f(x)]^n$ for positive integer n

Even More Special Limits and Laws

8. $\lim_{x\to a} x^n = a^n$ for positive integer n

9. $\lim_{x\to a} x^{1/n} = \lim_{x\to a} \sqrt[n]{x} = \sqrt[n]{a}$ for positive integer n and if n is even, $a \geq 0$

10. $\lim_{x\to a} [f(x)]^{1/n} = \lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$ for positive integer n. [In the case that n is even, $f(x) \geq 0$]

Ex: Evaluate the following limit, justifying each step with limit laws.

$$\lim_{x\to 2} \frac{3x^2+2x+2}{\sqrt{2x-1}} = \frac{\lim_{x\to 2} 3x^2+2x+2}{\lim_{x\to 2} \sqrt{2x-1}} \qquad \text{by Law } 6$$

$$= \frac{\lim_{x\to 2} 3x^2 + \lim_{x\to 2} 2x + \lim_{x\to 2} 2}{\sqrt{\lim_{x\to 2}(2x-1)}} \qquad \text{by Law } 3 \,\&\, 10$$

$$= \frac{3\lim_{x\to 2} x^2 + 2\lim_{x\to 2} x + \lim_{x\to 2} 2}{\sqrt{2\lim_{x\to 2} x - \lim_{x\to 2} 1}} \qquad \text{by Law } 3 \,\&\, 4$$

$$= \frac{3(2)^2+2(2)+2}{\sqrt{2(2)-1}} \qquad \text{by Law } 8, 2, \&\, 1$$

To find $\lim_{x \to a} f(x)$, we can use direct substitution (i.e., plug $x = a$ into $f(x)$

when the function is "nice" at $x = a$.

Strategy for Finding Limits $\left[\text{i.e. Evaluating } \lim_{x \to a} f(x) \text{ analytically}\right]$

★ Always Try Direct Substitution First ★

1. If you get a finite real number, you're done. $\lim_{x \to a} f(x) = f(a)$.

2. If you get $\frac{nonzero\#}{0}$, then it is an infinite limit.

3. If you get $\frac{0}{0}$, this is called an INDETERMINATE FORM

(i.e. unable to determine <u>YET</u>!!!) $\Rightarrow$ More Work!!!

<u>**Ex**</u> $\lim\limits_{x\to 4} \dfrac{x^2-16}{x-4} = \dfrac{0}{0}$

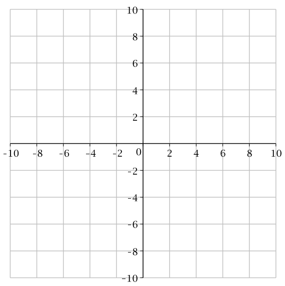
Note: $f(x) = \dfrac{x^2-16}{x-4}$

(a). What is the domain of $f(x)$?

But is there an asymptote at $x = 4$?

(b). Use your calculator to graph the function and sketch it below.

Is the graph what you expected?



(c). From your graph, determine $\lim\limits_{x\to 4} \dfrac{x^2-16}{x-4}$

(d). Observe:

$$f(x) = \frac{x^2-16}{x-4} = \frac{(x-4)(x+4)}{(x-4)}$$