

Books, notes (in any form), calculators, etc., are not allowed. You must *show all your work* for full credit. Good Luck!

1. (15 pts) Given the following function and its derivatives

$$f(x) = \frac{-2x^2}{x^2 + 3}$$

$$f'(x) = \frac{-12x}{(x^2 + 3)^2}$$

$$f''(x) = \frac{36(x^2 - 1)}{(x^2 + 3)^3}$$

(a). Fill in the following information about the function and its graph. Show all work and write "none", if applicable.

domain:

All real #s

$$x^2 + 3 = 0 \text{ Never}$$

x-intercept(s):

pt. (0,0)

$$x\text{-int}(y=0): -2x^2 = 0 \Rightarrow x^2 = 0 \Rightarrow x = 0$$

$$y\text{-int}(x=0): f(0) = \frac{0}{3} = 0$$

$$V.A. x^2 + 3 = 0 \text{ Never.}$$

y-intercept:

pt. (0,0)

$$H.A. \lim_{x \rightarrow \infty} \frac{-2x^2}{x^2 + 3} = -2 \quad \lim_{x \rightarrow -\infty} \frac{-2x^2}{x^2 + 3} = -2$$

vertical

asymptote(s):

None

$$f'(x) = 0$$

$$f'(x) \text{ DNE}$$

$$-12x = 0 \\ x = 0$$

$$(x^2 + 3)^2 = 0$$

$$x^2 + 3 = 0 \text{ Never.}$$

$$y = -2 \text{ is H.A.}$$

horizontal

asymptote(s):

$y = -2$

slant asymptote:

None

critical numbers:

$x = 0$

f'



intervals where increasing:

$(-\infty, 0)$

$$x = -10 \quad 0 \quad x = 10$$

Local max @ $x = 0 \Rightarrow f(0) = 0$

intervals where decreasing:

$(0, \infty)$

coordinates of local max(s):

$(0, 0)$

coordinates of local min(s):

None

intervals where concave up:

$(-\infty, -1) \cup (1, \infty)$

intervals where concave down:

$(-1, 1)$

$$f''(x) = 0$$

$$36(x^2 - 1) = 0$$

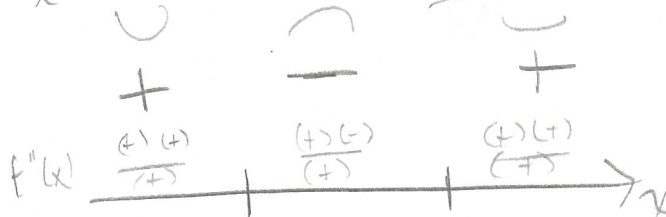
$$x^2 = 1$$

$$x = \pm 1$$

$$f''(x) \text{ DNE}$$

$$(x^2 + 3)^3 = 0$$

$$x^2 + 3 = 0 \text{ Never}$$



Inflection Point(s):

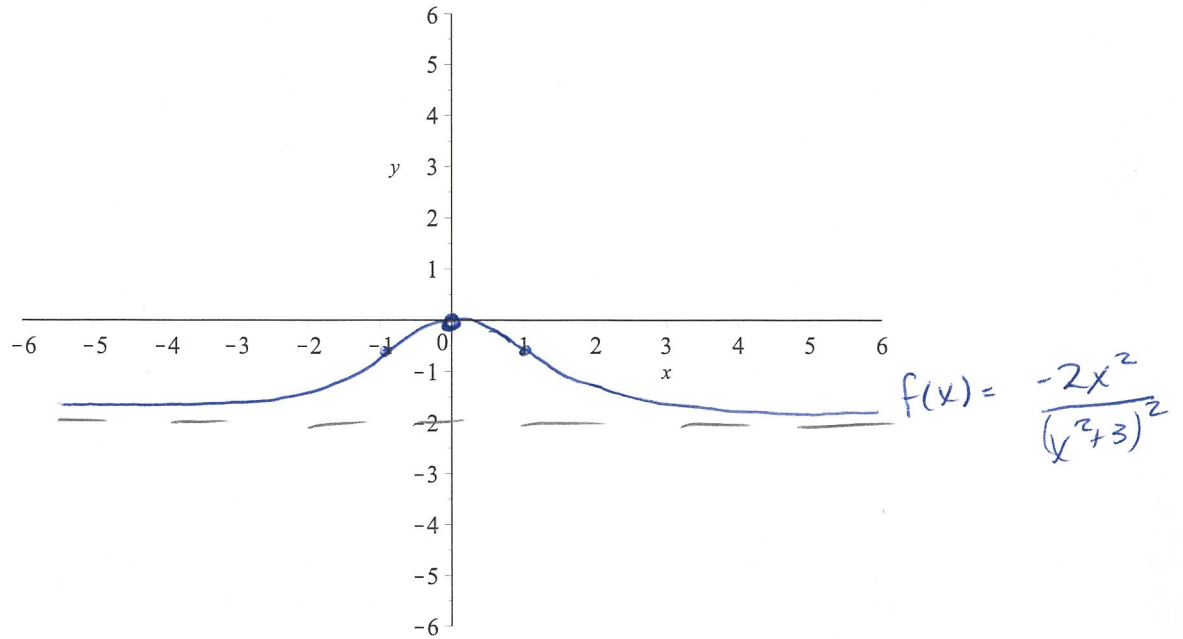
$(1, -\frac{1}{2})$ and $(-1, -\frac{1}{2})$

PoI @ $x = -1$ and $x = 1$

$$f(1) = \frac{-2(1)^2}{1+3} = \frac{-2}{4} = -\frac{1}{2}$$

$$f(-1) = \frac{-2(-1)^2}{(-1)^2 + 3} = \frac{-2}{4} = -\frac{1}{2}$$

- (b). Sketch the graph of the function on the set of axes provided. Label any maximum and minimum values and inflection points.



Just an extra set of axes, in case you need it.

